Determining the jet transport coefficient $\hat{q}$ of the quark-gluon plasma using Bayesian parameter estimation

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RHIP Seminar
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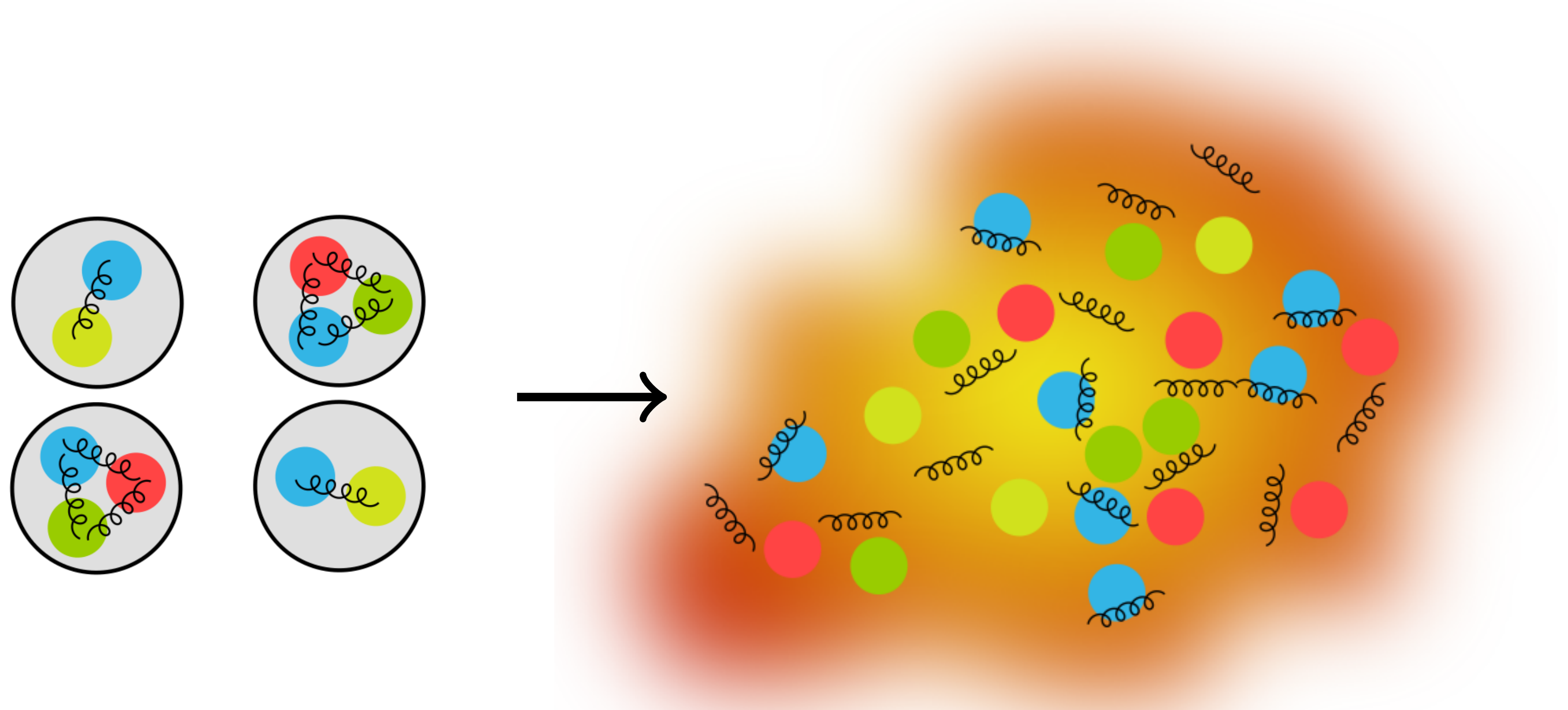
Jet quenching in the quark-gluon plasma

We would like to learn big questions about the deconfined state of QCD

- ☐ What are the relevant degrees of freedom of the QGP?
  - ☐ Quasi-particles?
- ☐ How does a strongly-coupled system arise from QFT?
  - ☐ Compute bulk properties from first principles?



W. Ke

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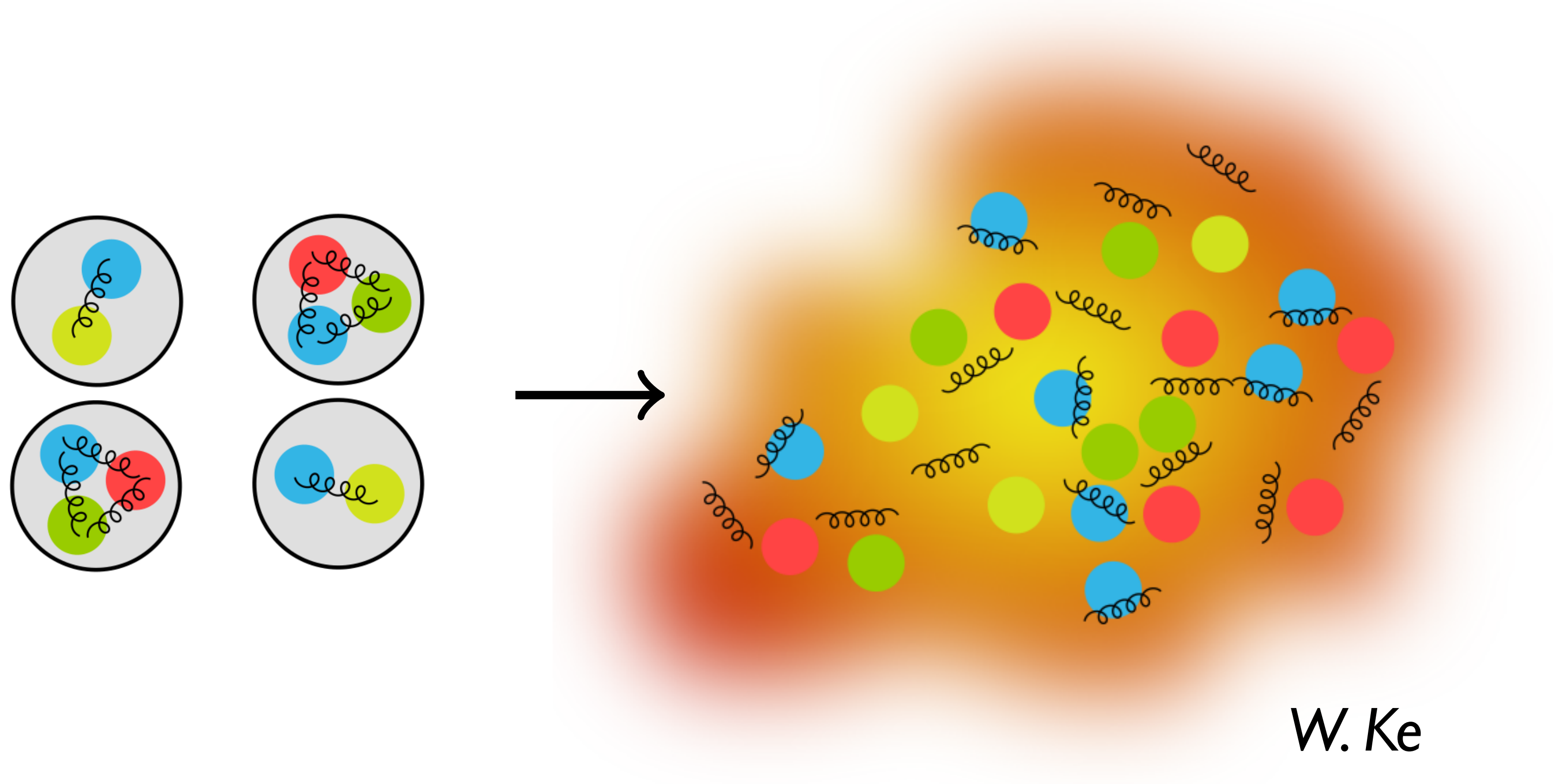

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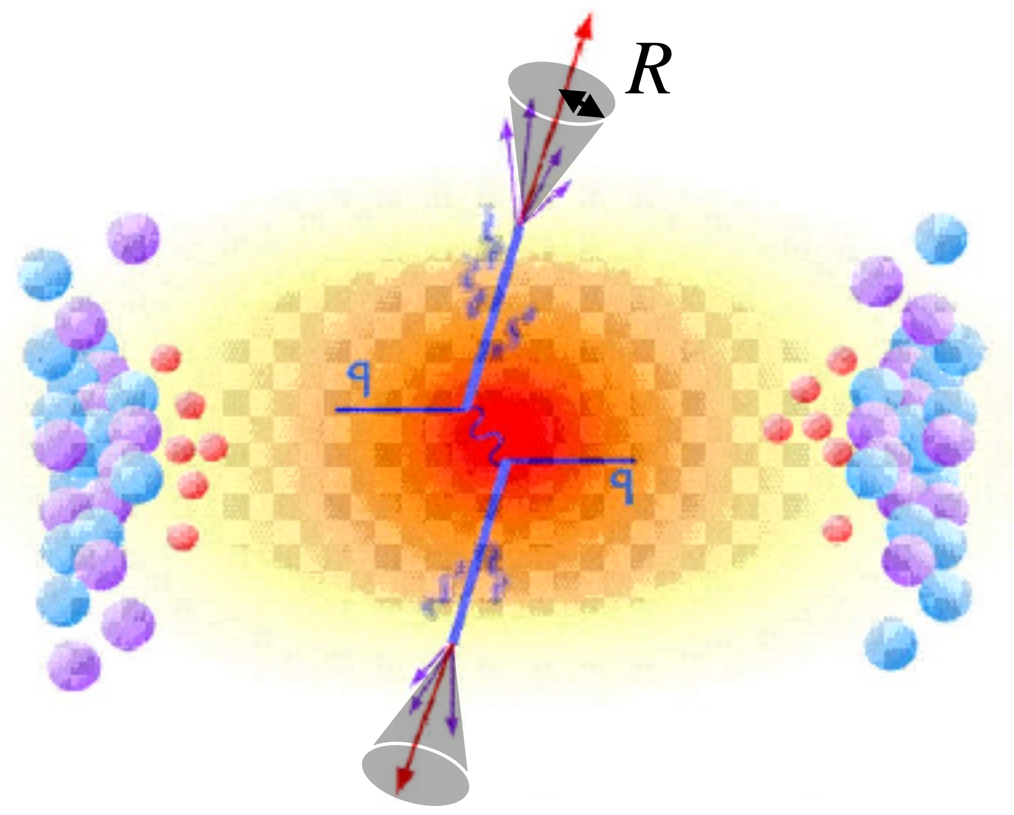
Jets offer a compelling tool

- Jets can probe from the smallest medium scales to the largest medium scales
- Jet evolution can be computed from first principles
- Jets are strongly sensitive to (some) medium properties: $\hat{q}$



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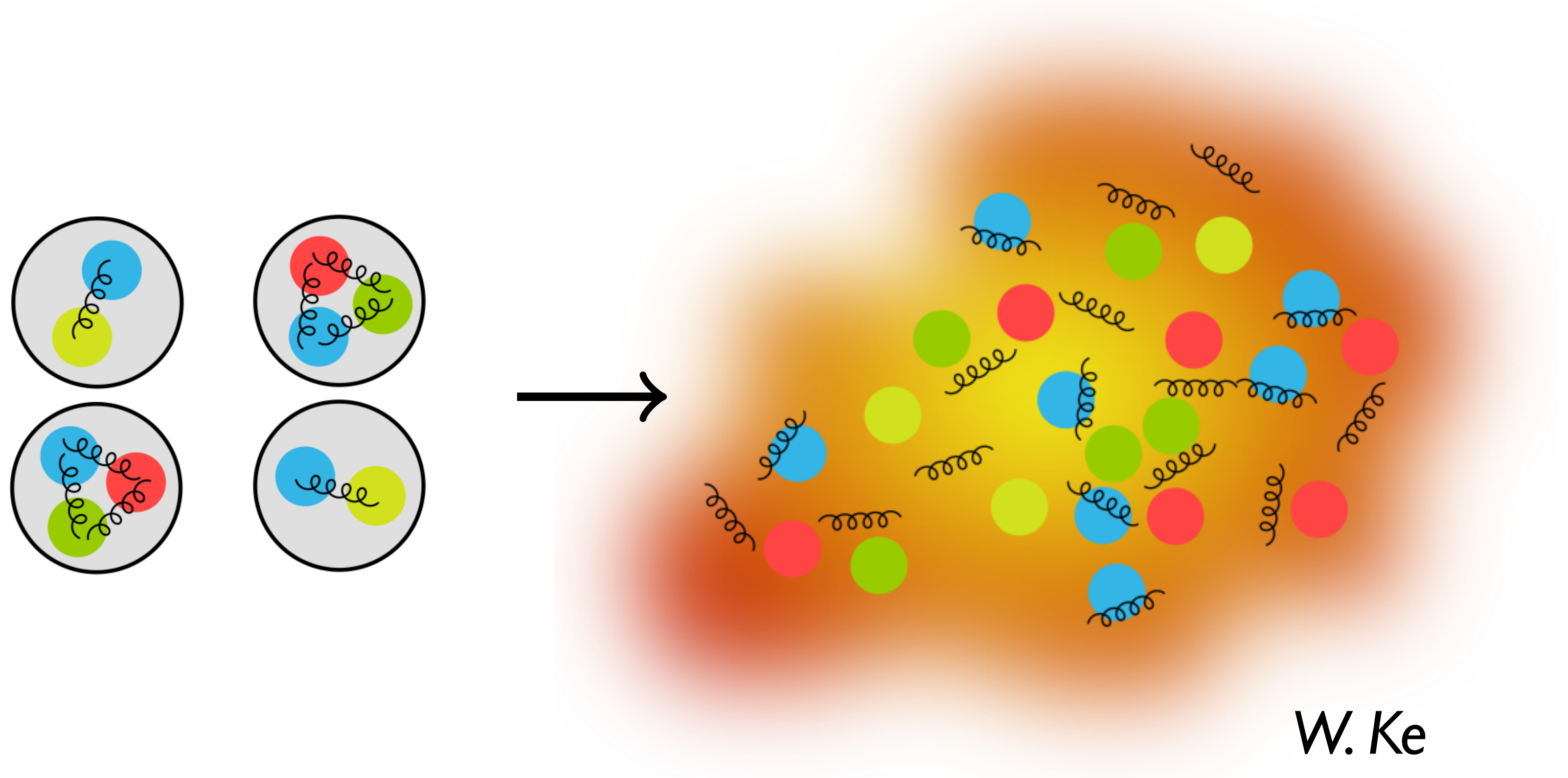

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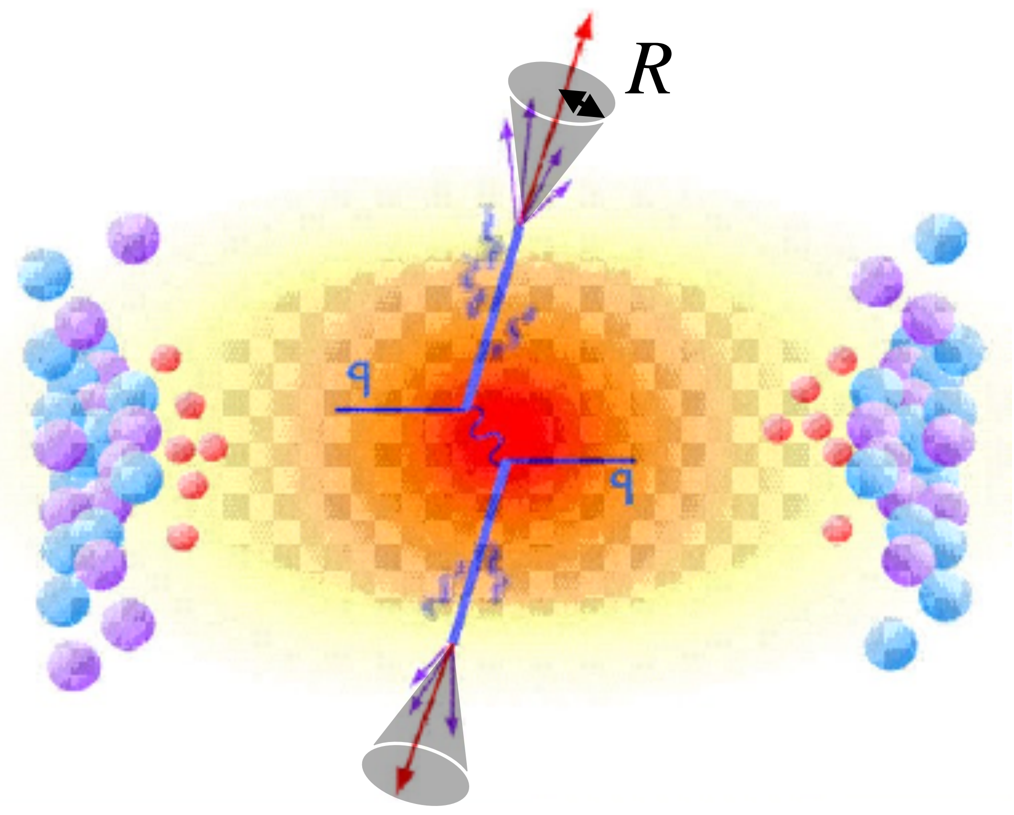
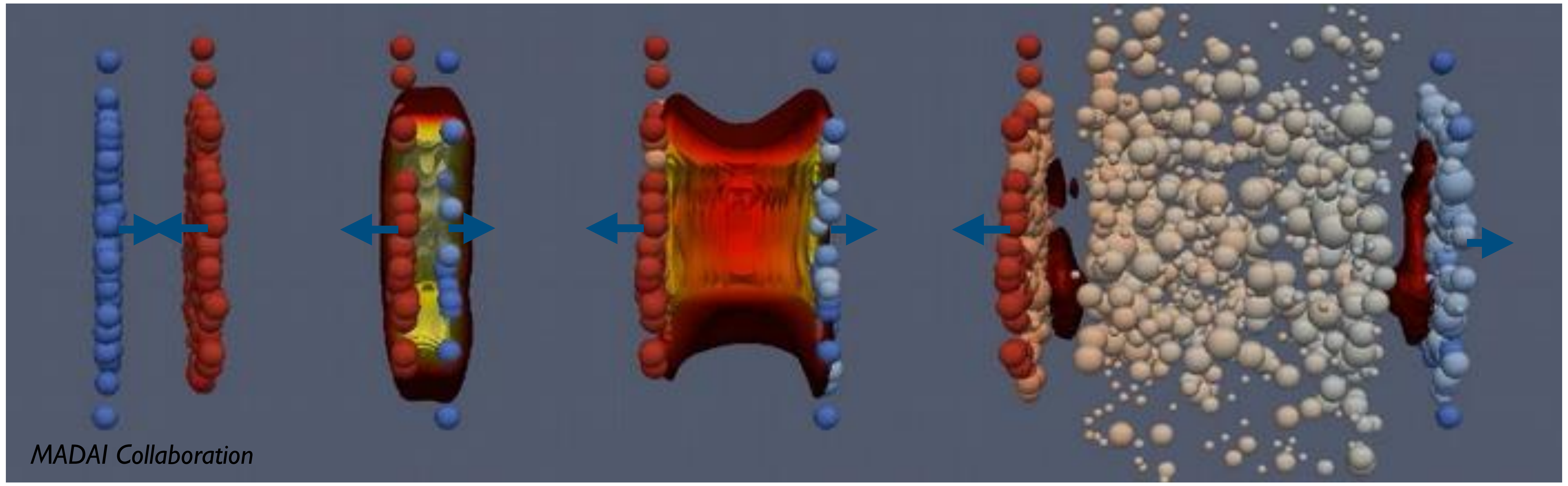


However, it is clear by now that this endeavor is not simple…

Challenge #1

Jet evolution involves physics that is not known from first principles: initial state, hydrodynamic evolution, medium response, hadronic rescattering, hadronization



MADAI Collaboration

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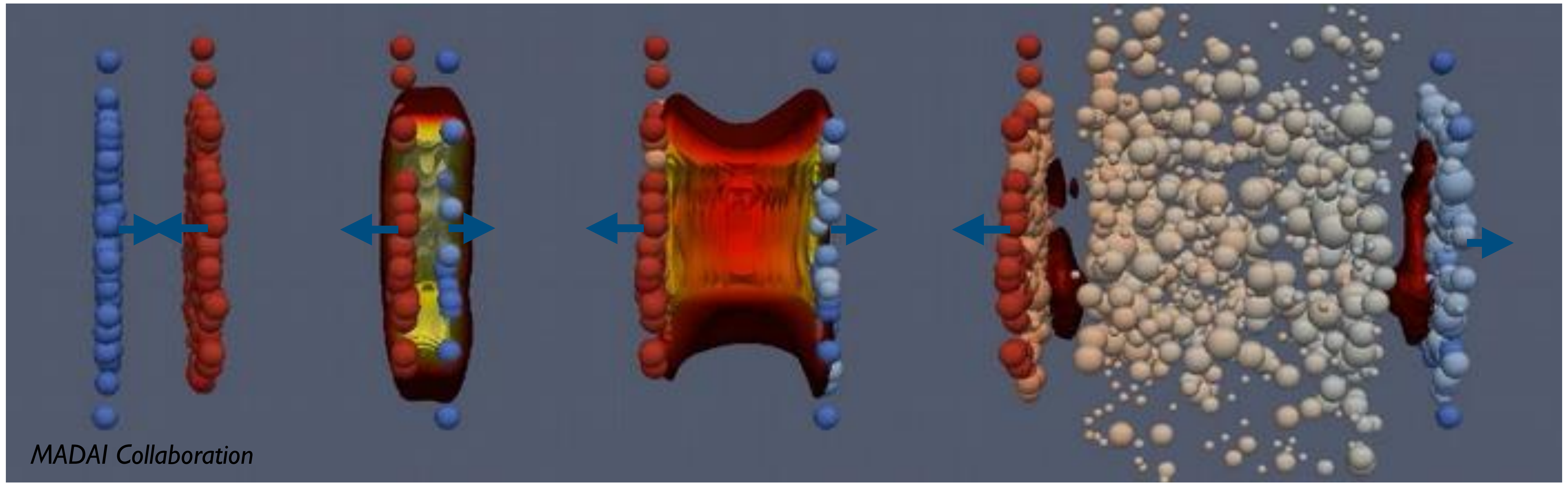
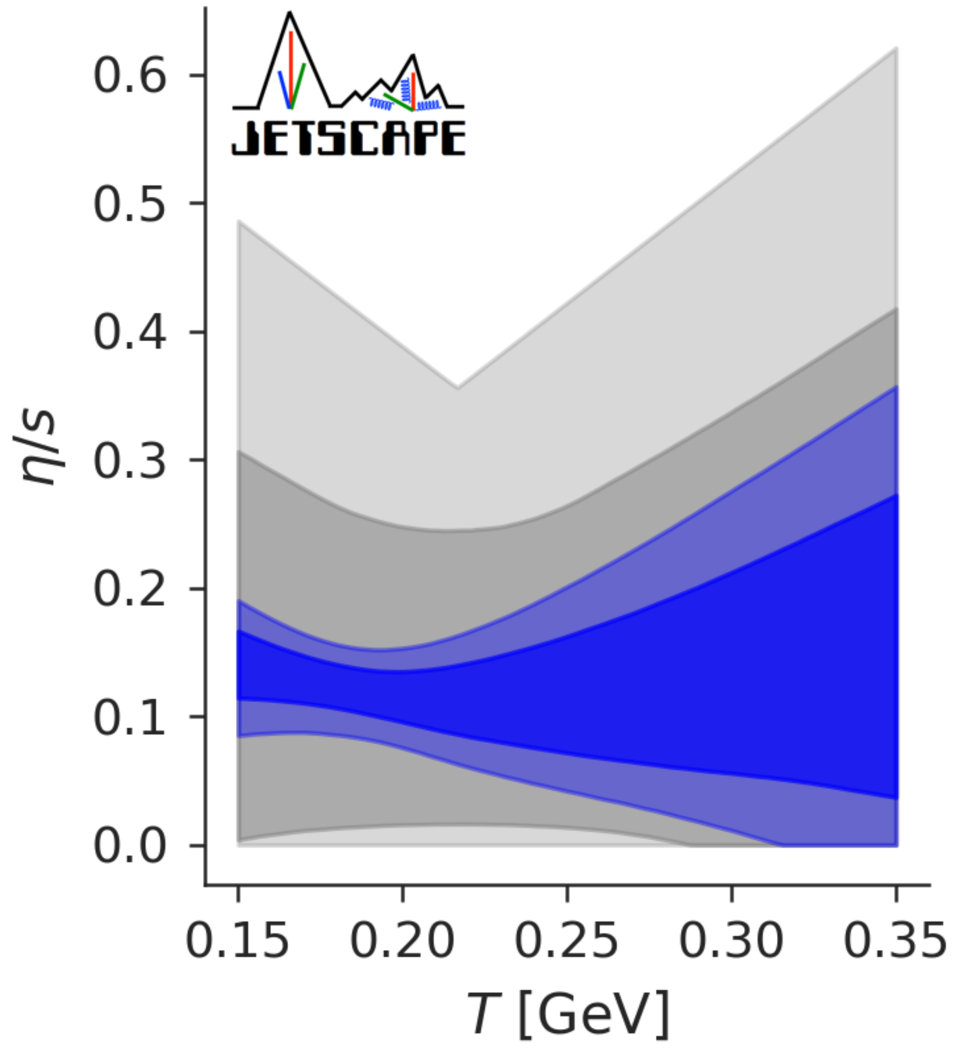


JETSCAPE, 2011.01430



Global analysis is needed to fit models of the physics that are not known from first-principles

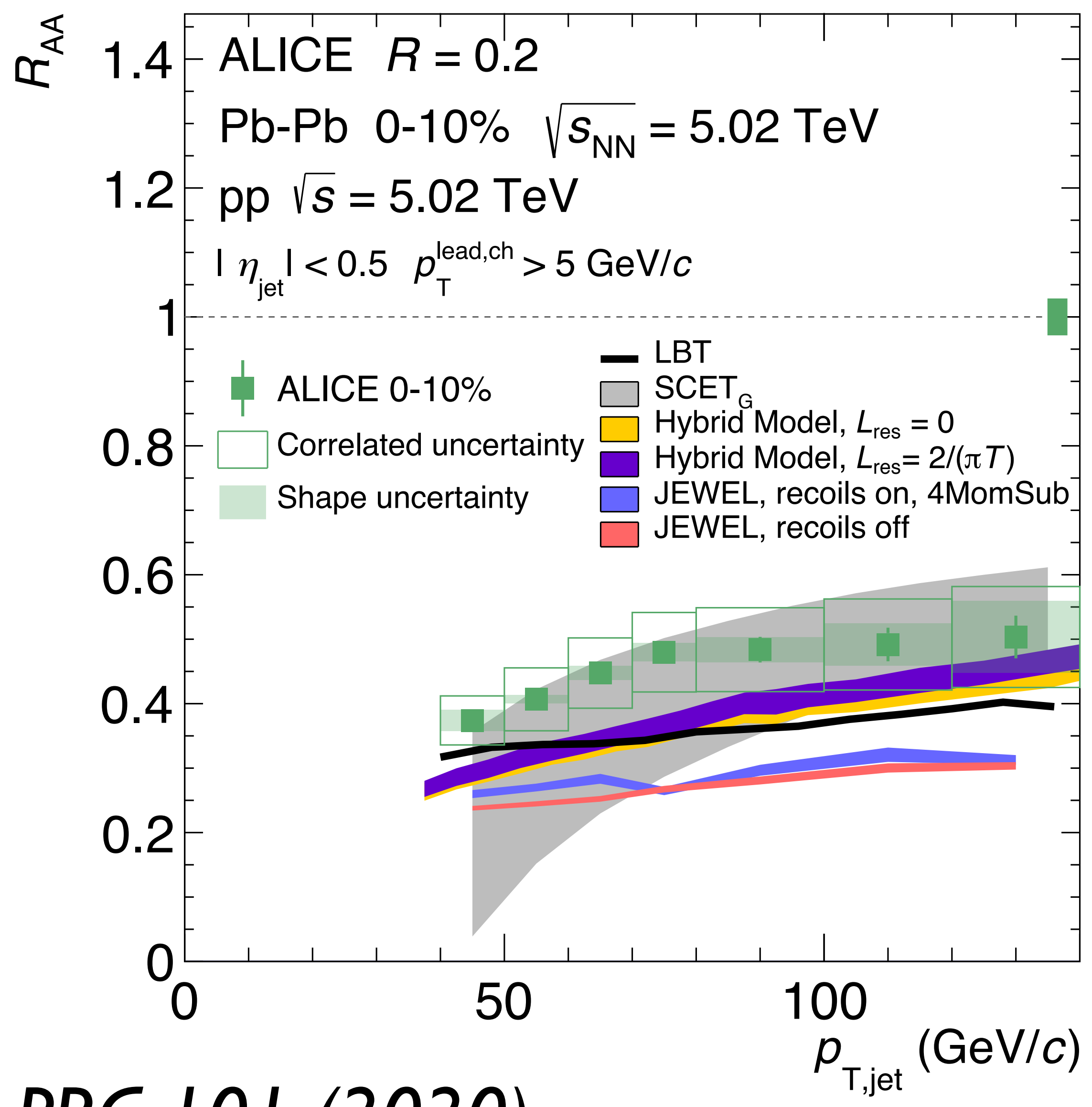
Challenge #2

Jet evolution itself is complicated, and there is no (known) golden observable

Simultaneous unknowns in jet quenching theory:

- ☐ Strongly-coupled vs. weakly-coupled interaction
- ☐ Spacetime picture of parton shower
- ☐ Factorization breaking
- ☐ Color coherence
- ☐ …

Jet R_{AA}, for example, is strongly modified — but models with different physics predict similar values



ALICE, PRC 101 (2020)

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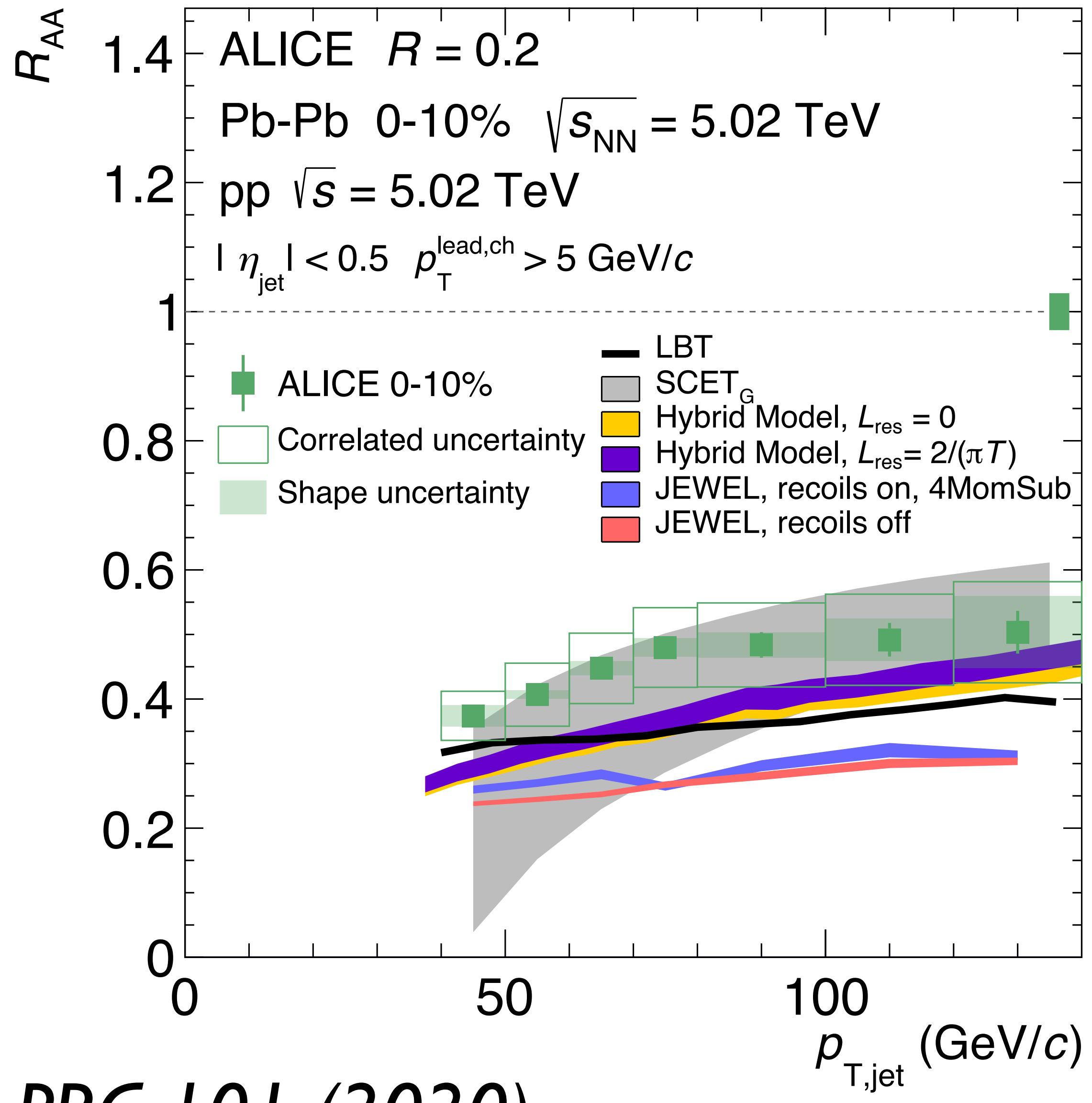


ALICE, PRC 101 (2020)

Need global analysis of multiple jet observables to:

1. **Distinguish theoretical approaches**
2. **Precisely determine medium properties**

→ **This talk**

Outline

$\hat{q}$ and JETSCAPE

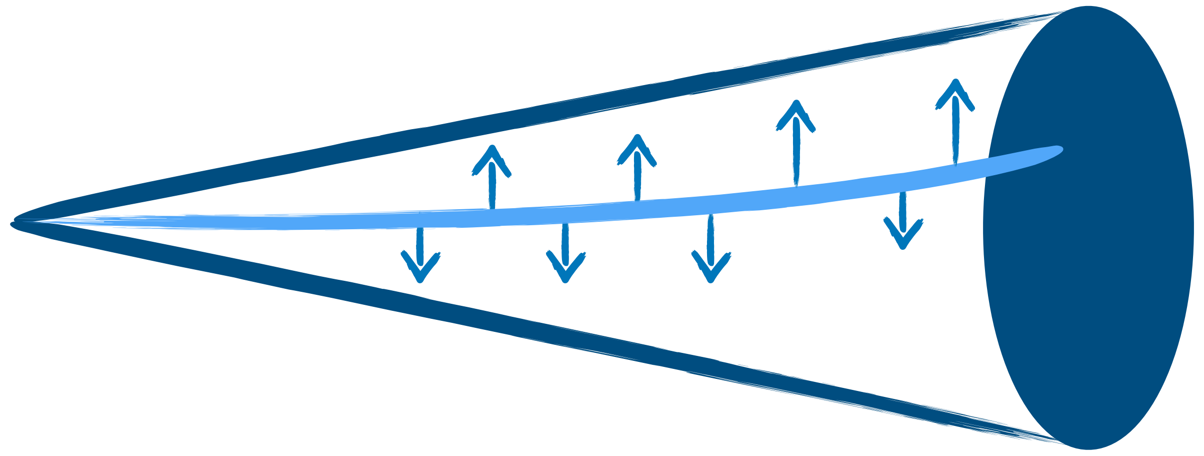
Bayesian parameter estimation

Results

The jet transverse diffusion coefficient

As a parton propagates through the QGP, it will undergo momentum exchanges transverse to its direction of propagation:

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{L} \int dk_\perp^2 \frac{dP\left(k_\perp^2\right)}{dk_\perp^2}$$



where $P\left(k_\perp^2\right)$ is a scattering kernel.

The accumulated $\langle k_\perp^2 \rangle$ can arise from various microscopic interactions:

- ☐ Single hard emission
- ☐ Multiple soft scattering
- ☐ Smooth drag

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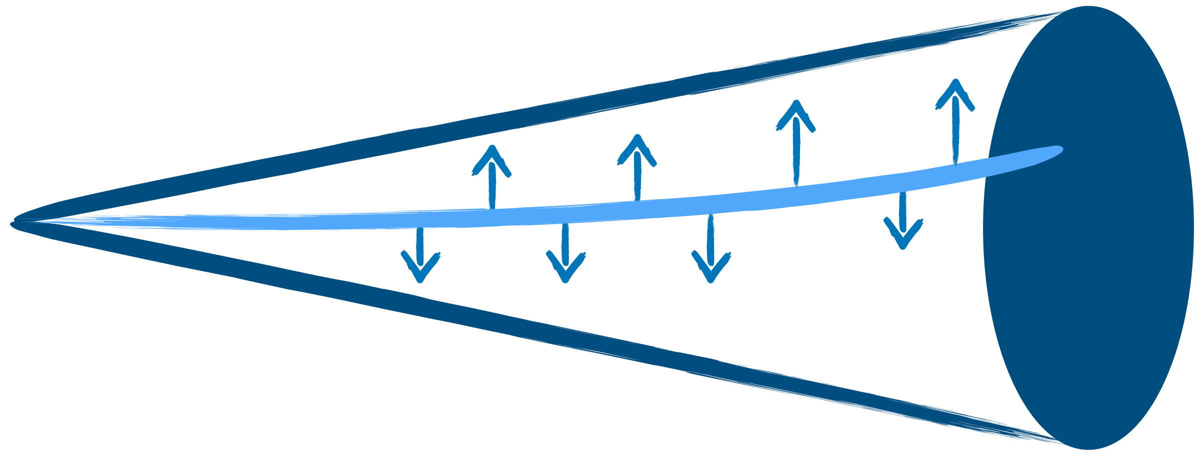

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$\hat{q}$ is one of the most important quantities characterizing jet quenching

- ☐ Out-of-cone transport — "energy loss"
  In BDMPS: $\Delta E \sim \hat{q}L^2$
- ☐ Broadening
  In BDMPS: $\Delta\varphi \sim \sqrt{\hat{q}L}$

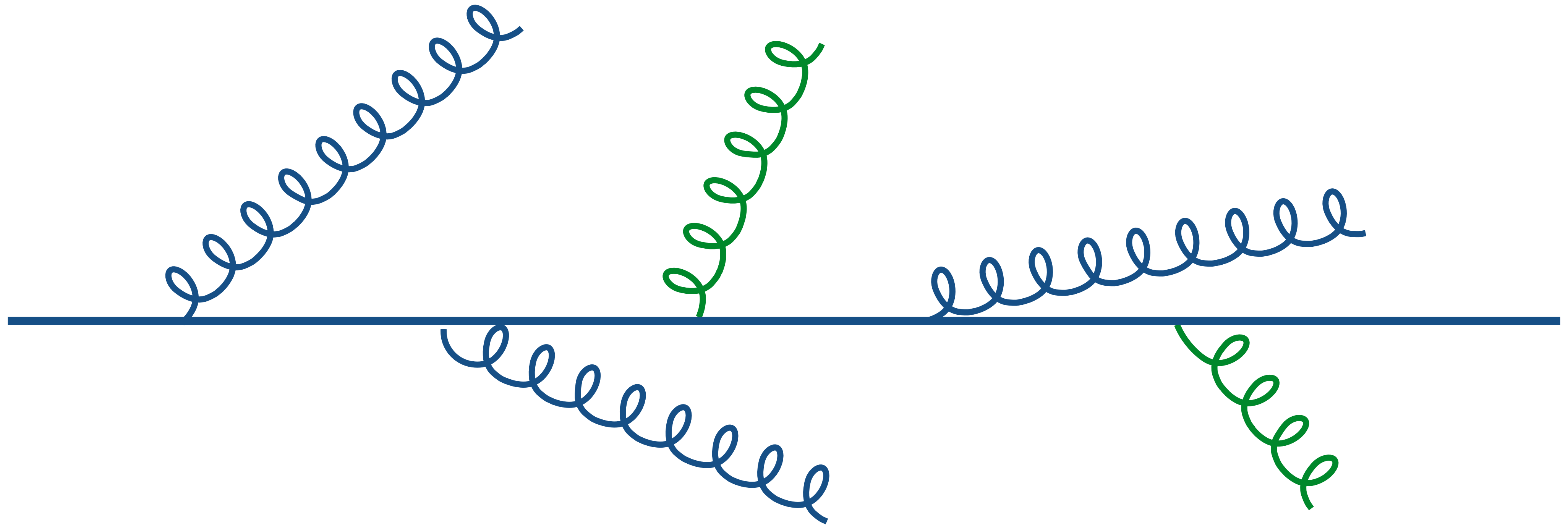
Leading hadrons

While $\hat{q}$ is important for all jet observables, it is not the **only** important physics

- ☐ Re-scattering of soft emissions
- ☐ Medium response

→ Relevant to reconstructed jets

For leading hadron p_{T}, however, $\hat{q}$ is the dominant physics



We only need to know what is radiated away from the leading parton

— not what happens to those radiations

Calculating $\hat{q}$

Under certain assumptions, $\hat{q}$ can be calculated

For example, assuming perturbative, small-angle elastic scattering off a thermal medium:

HTL formula

$$\hat{q}\,(T,E) = C_R \frac{42\zeta(3)}{\pi} \alpha_S^2\, T^3\ \ln\left[\frac{\mu^2}{6\pi\alpha_S T^2}\right]$$

Parton-medium interaction scale

Local temperature

Parton energy

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However, we will instead **parameterize** $\hat{q}$ in JETSCAPE with a more general form:

$$\frac{\hat{q}\,(E,T)\,|_{A,B,C,D}}{T^3} = 42 C_R \frac{\zeta(3)}{\pi} \left(\frac{4\pi}{9}\right)^2 \left\{ \frac{A\left[\ln\left(\frac{E}{\Lambda}\right) - \ln(B)\right]}{\left[\ln\left(\frac{E}{\Lambda}\right)\right]^2} + \frac{C\left[\ln\left(\frac{E}{T}\right) - \ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2} \right\}$$

High-virtuality inspired

T-independent

HTL-inspired

elastic scattering off temperature T

MATTER

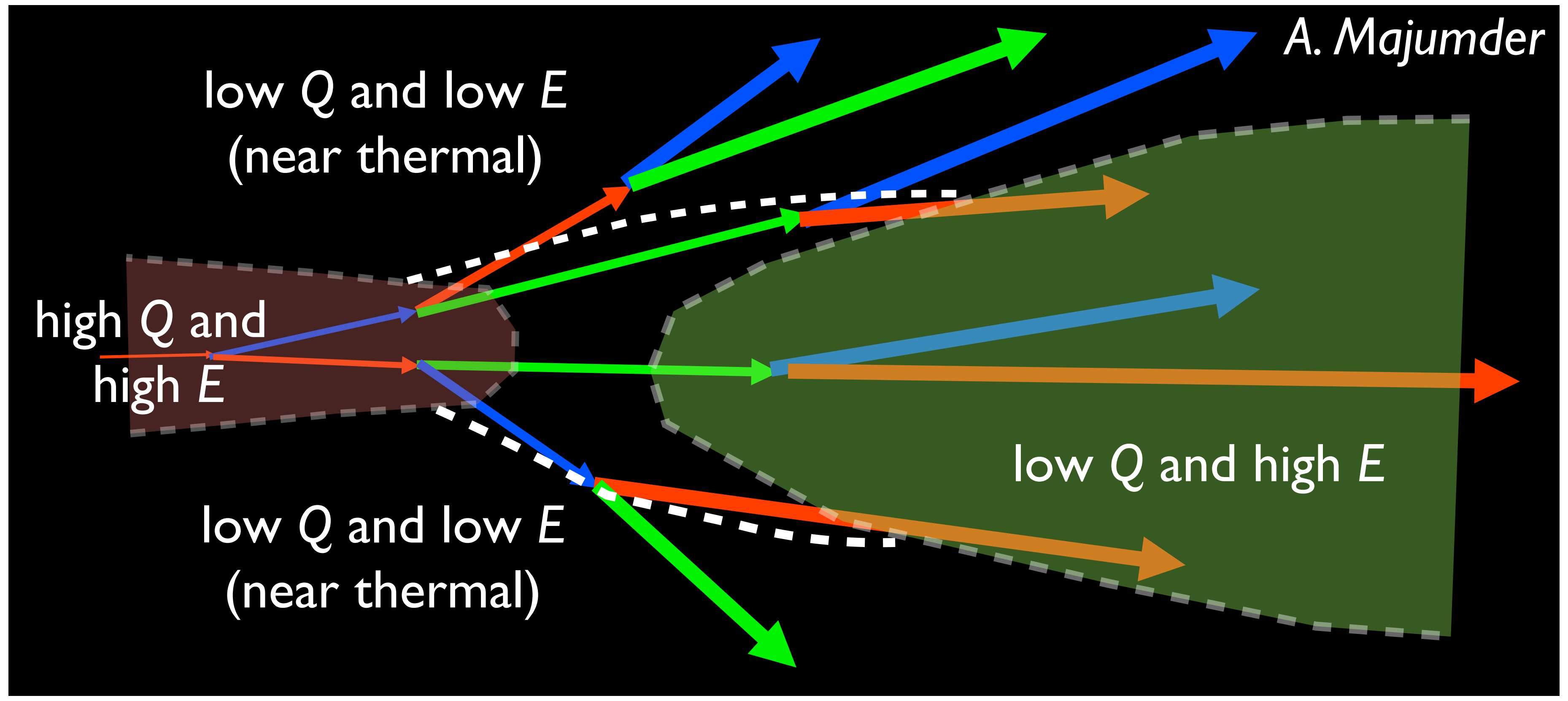
Majumder PRC 88 (2013)
Cao, Majumder PRC 101 (2020)

Medium-modified splitting function

$$P_a(z,Q^2) = P_a^{\mathrm{vac}}(z) + P_a^{\mathrm{med}}(z,Q^2)$$

Virtuality-ordered shower modified
to contain spacetime information

High-virtuality, radiation-dominated regime: $Q \gg \sqrt{\hat{q}E}$



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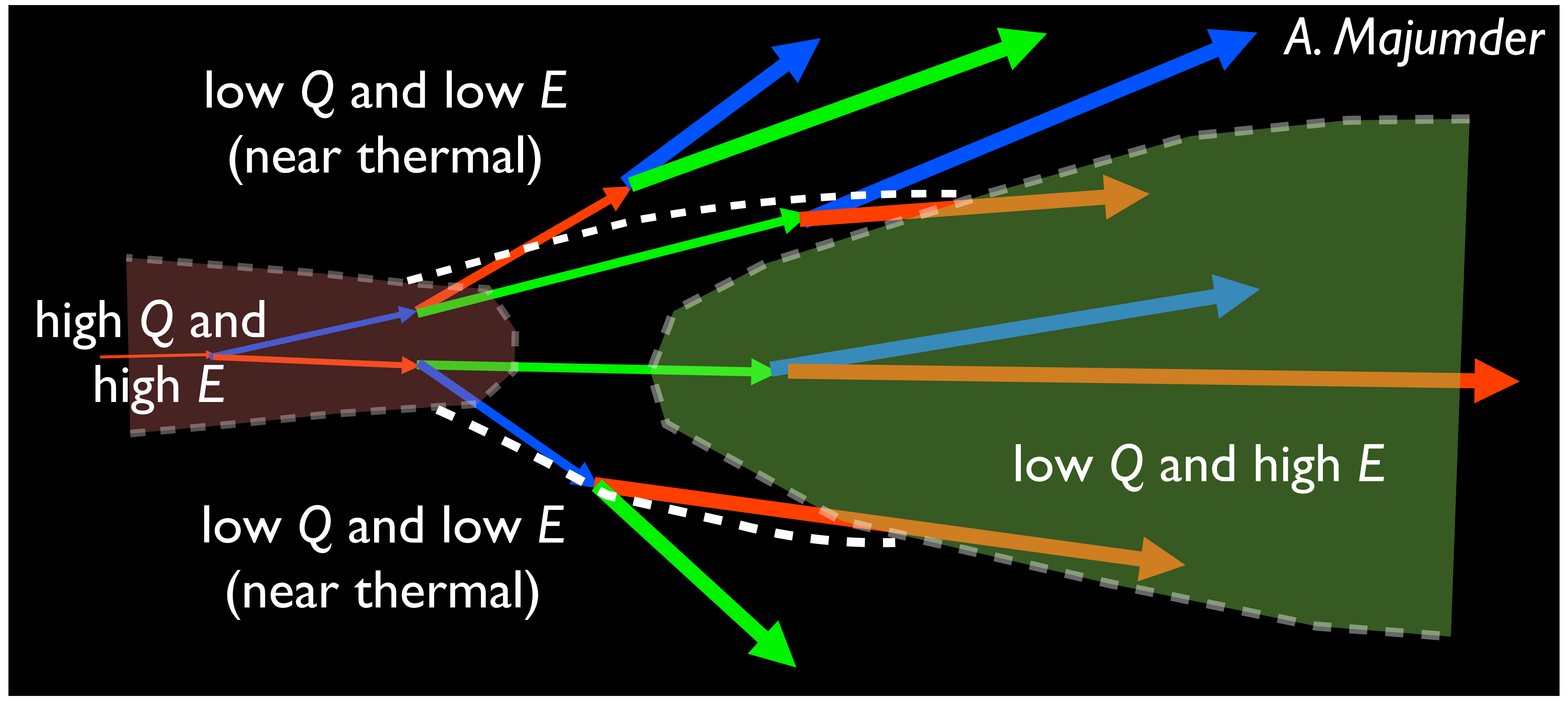


LBT

Cao, Luo, Qin, Wang PRC 94 (2016)
PLB 777 (2018)

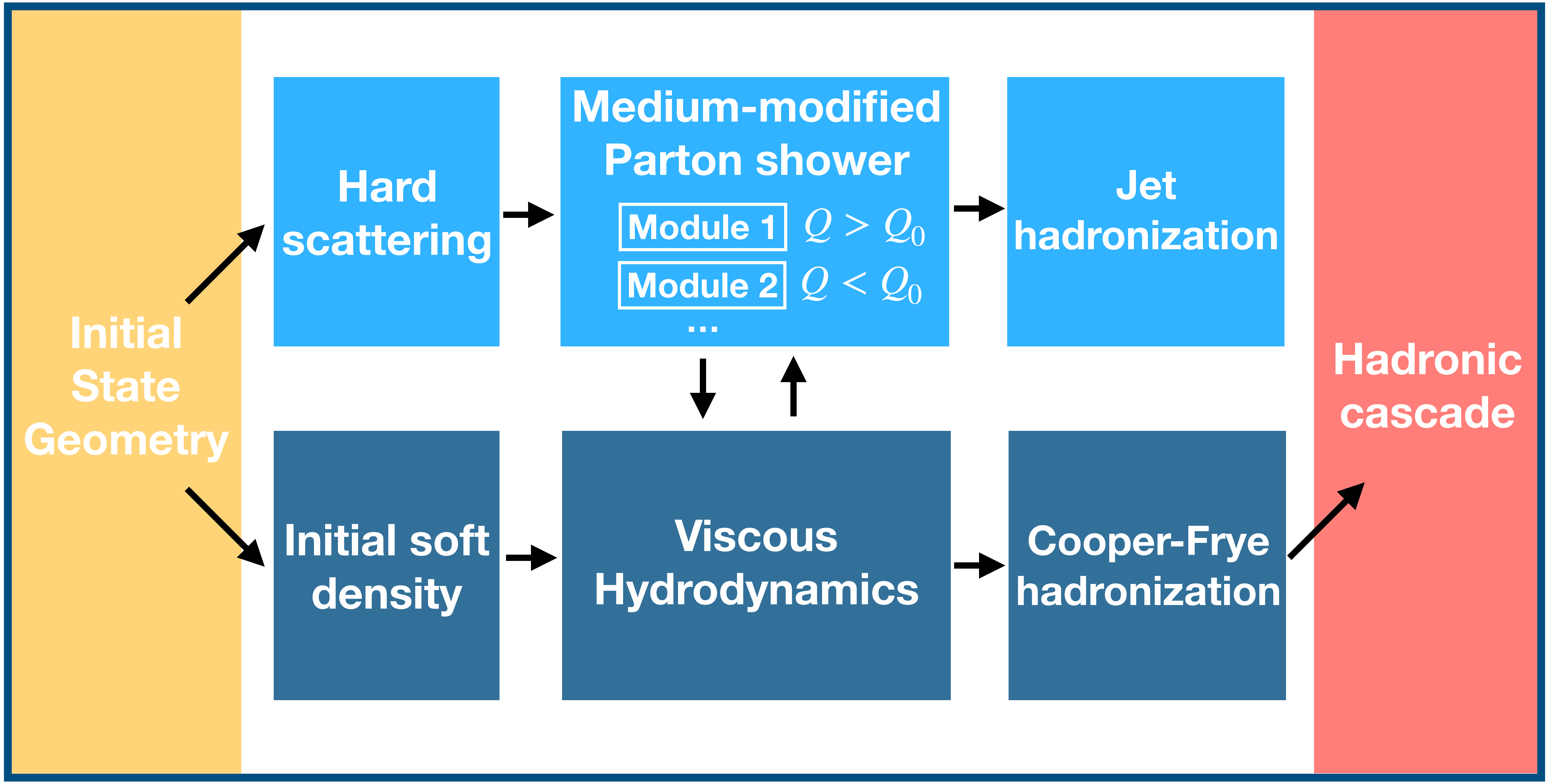
Elastic and inelastic scatterings — linearized Boltzmann transport of jet partons

Inelastic scatterings generate gluon radiation — higher twist formalism

Low-virtuality, scattering-dominated regime

Broadening due to elastic scattering

Jet quenching in JETSCAPE



Initial State Geometry

Hard scattering

Medium-modified Parton shower

Module 1 $Q > Q_0$

Module 2 $Q < Q_0$

...

Jet hadronization

Initial soft density

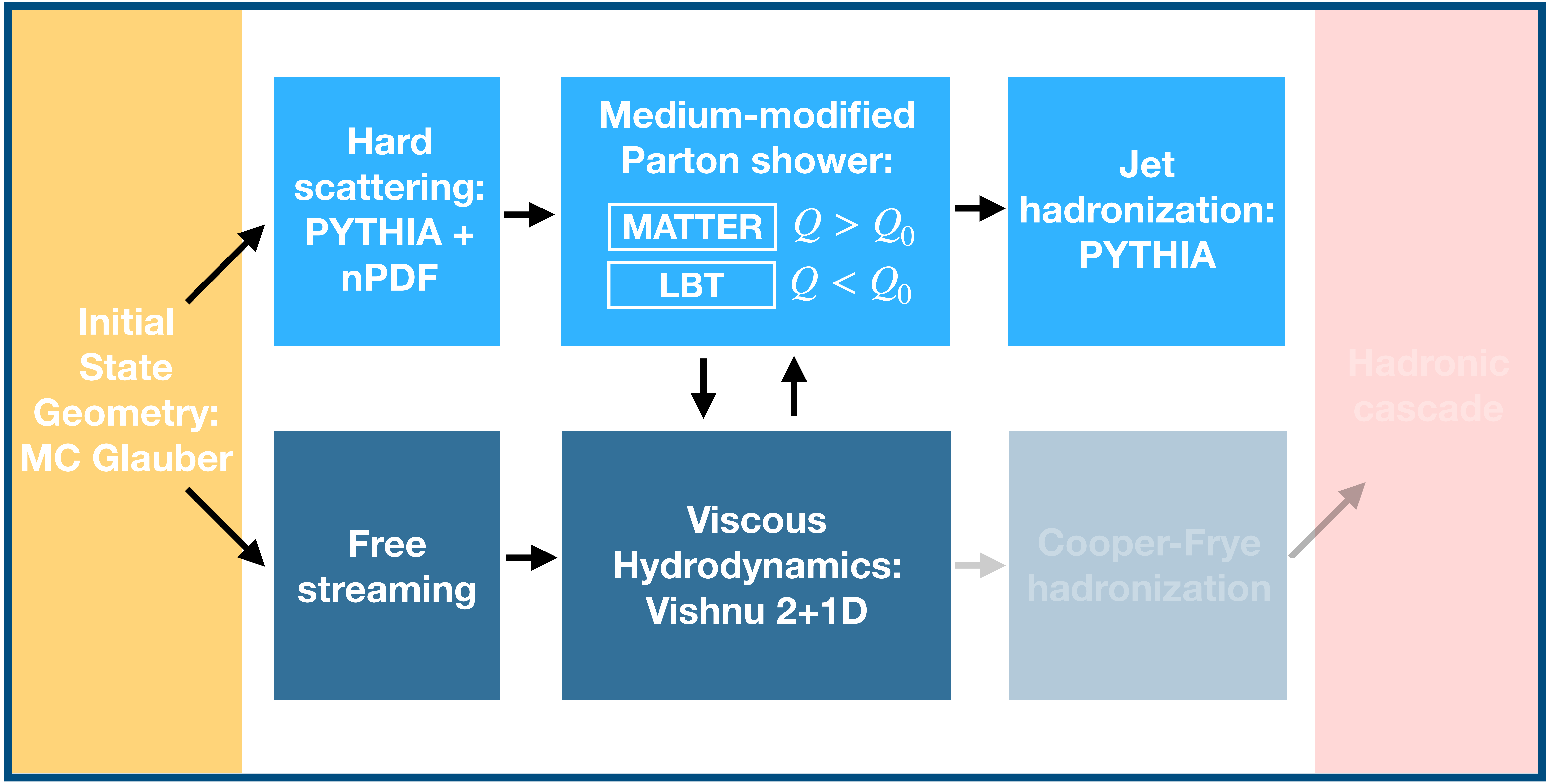
Viscous Hydrodynamics

Cooper-Frye hadronization

Hadronic cascade

This talk

$$\hat{q}(E,T)\Big|_{\theta=\{A,B,C,D\}}$$



Initial State Geometry: MC Glauber

Hard scattering: PYTHIA + nPDF

Medium-modified Parton shower:

MATTER $Q > Q_0$

LBT $Q < Q_0$

Jet hadronization: PYTHIA

Free streaming

Viscous Hydrodynamics: Vishnu 2+1D

Cooper-Frye hadronization

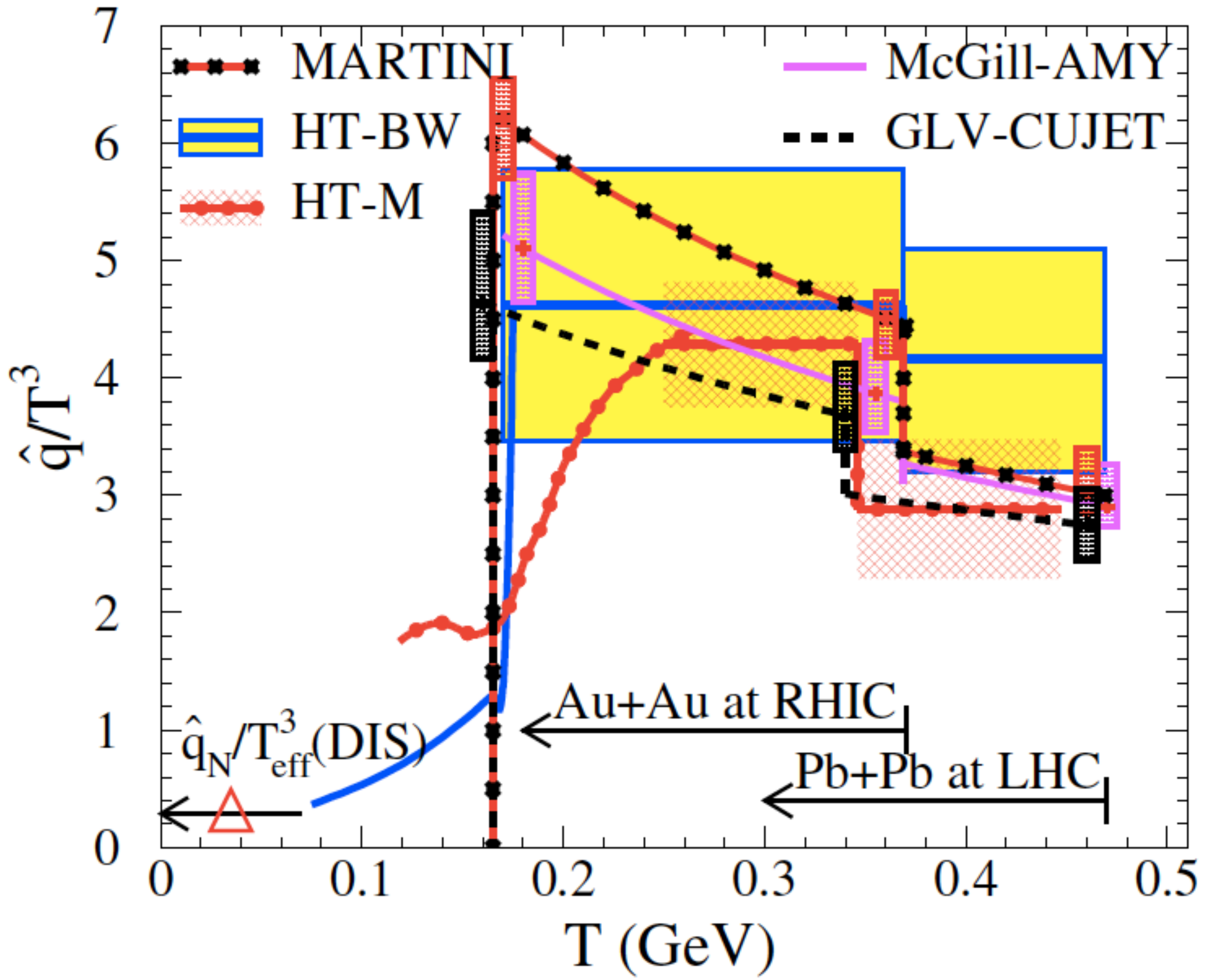
Hadronic cascade

JET Collaboration

Previous work: Separate fits of $\hat{q}$ at RHIC and LHC for various pQCD models

Improvements in this talk:

- ☐ Extraction of $\hat{q}$ as a continuous function of T, E
- ☐ Bayesian statistics — correct approach
- ☐ Improved theoretical models



JET Collaboration, PRC 90 (2014)

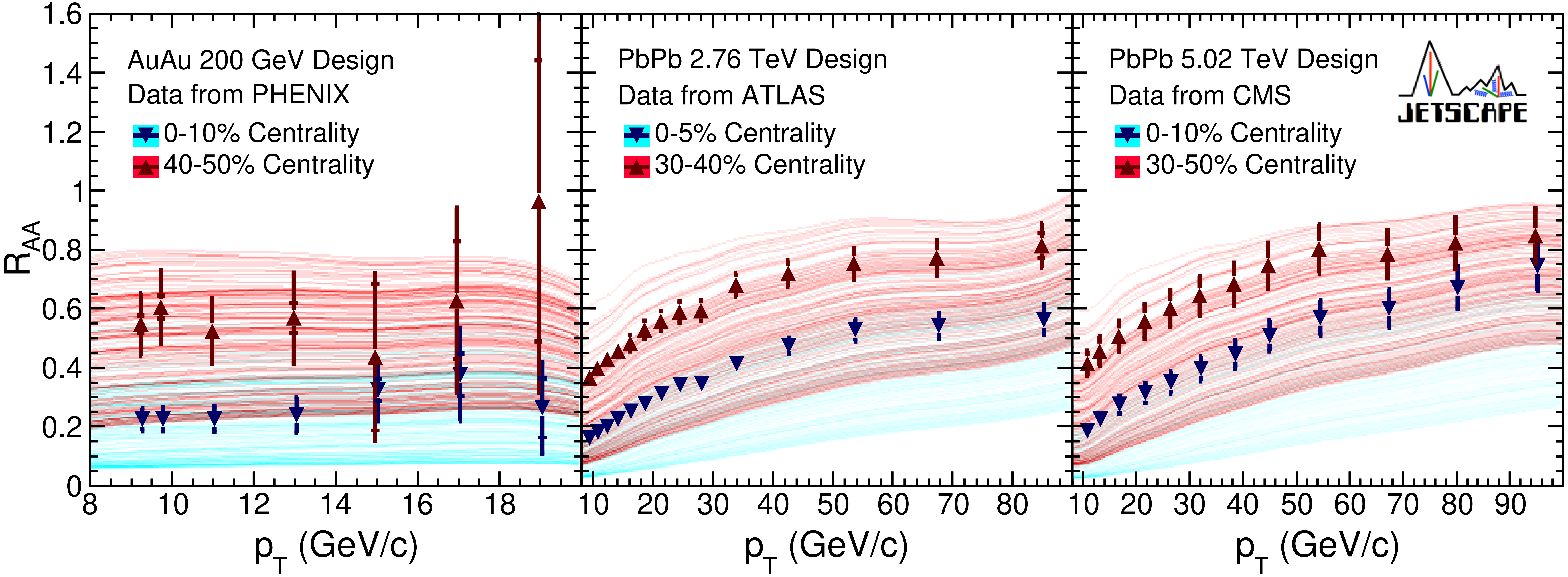
Experimental data

PHENIX PRC 87 (2013)
CMS EPJC 72 (2012)
ATLAS JHEP 09 (2015)

Inclusive hadron R_{AA}

- Central + semi-central
- One dataset per $\sqrt{s_{\mathrm{NN}}} = 0.2,\ 2.76,\ 5.02\ \mathrm{TeV}$
- Truncate at $p_{\mathrm{T}} > 8\ \mathrm{GeV}/c$ to avoid medium-modified hadronization effects

→ Vary T



Experimental uncertainties

We decompose the experimental covariance matrix into several sources:

- Uncorrelated uncertainties — e.g. statistical
- Luminosity uncertainty — fully correlated across p_{T}, centrality bins
- T_{AA} uncertainty — fully correlated across p_{T} bins
- Other unspecified systematic uncertainties

$$\Sigma_k^E = \Sigma_k^{\mathrm{uncorr}} + \Sigma_k^{\mathrm{fcorr}} + \Sigma_k^{\mathrm{lcorr}}$$

$$\Sigma_{k,ij}^{\mathrm{uncorr}} = \sigma_{k,i}^{\mathrm{uncorr}} \sigma_{k,j}^{\mathrm{uncorr}} \delta_{ij}$$

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There is a simple practice that we (experimentalists) need to start doing:

Report signed systematic uncertainty breakdowns in HEPData

(or full covariances matrices)

Bayesian parameter estimation

Goal: Use experimental data to constrain the value of $\hat{q}\,(E,T)$

1. Parameterize $\hat{q}(E,T)\Big|_{\theta=\{A,B,C,D\}}$ in a jet quenching model
2. Explore the parameter space θ to find the most likely values of θ for that model to explain the experimental data

We specifically want to constrain the **probability distribution** of $\hat{q}$

→ **Bayesian analysis**

Bayesian parameter estimation

$$\hat{q}(E,T)\Big|_{\theta=\{A,B,C,D\}}$$

$$P(\theta\,|\,D) \sim P(D\,|\,\theta)P(\theta)$$

Posterior

Likelihood

Prior

Bayesian parameter estimation

$$\hat{q}(E,T)\Big|_{\theta=\{A,B,C,D\}}$$

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The **prior** is our initial knowledge of the parameters — we will take a flat prior

Bayesian parameter estimation

$$\left.\hat{q}(E,T)\right|_{\theta=\{A,B,C,D\}}$$

$$\underset{\substack{\nearrow \\ \text{Posterior}}}{P(\theta\,|\,D)} \sim \underset{\substack{\nearrow \\ \text{Likelihood}}}{P(D\,|\,\theta)}\underset{\substack{\nwarrow \\ \text{Prior}}}{P(\theta)}$$

The **prior** is our initial knowledge of the parameters — we will take a flat prior

The **likelihood** characterizes how likely we would be to observe the given data, given a set of parameters θ

$$P(D\,|\,\theta) \sim \exp\left[-\left(\Delta_i \Sigma_{ij}^{-1} \Delta_j\right)^2\right]$$

where $\Delta_i = R^{\theta}_{\mathrm{AA,i}} - R^{\mathrm{data}}_{\mathrm{AA}}$

Σ is the covariance matrix

Bayesian parameter estimation

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Σ is the covariance matrix

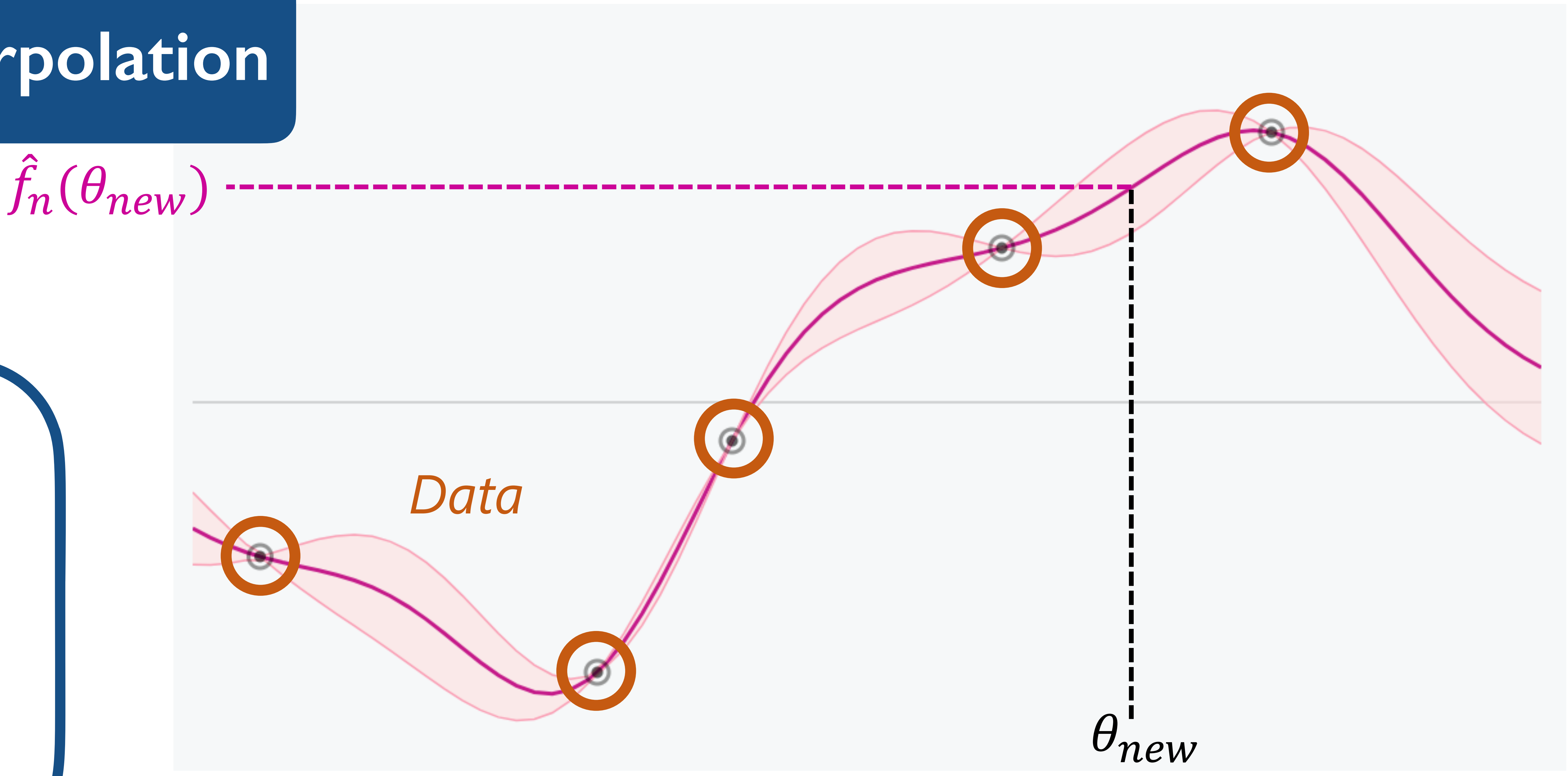
The **posterior** is what we want — probability distribution of $\hat{q}$, given the data

We will sample the posterior using Markov Chain Monte Carlo (MCMC)

Gaussian Process Emulators

In order to evaluate the likelihood across the parameter space θ, we need to know the R_{AA} predicted by JETSCAPE at **prohibitively many** different θ

Solution: Non-parametric interpolation



Simon Mak

This allows us to train an interpolator using $\mathcal{O}\left(10 \times \dim \theta\right)$ JETSCAPE model calculations

with quantification of interpolation uncertainty

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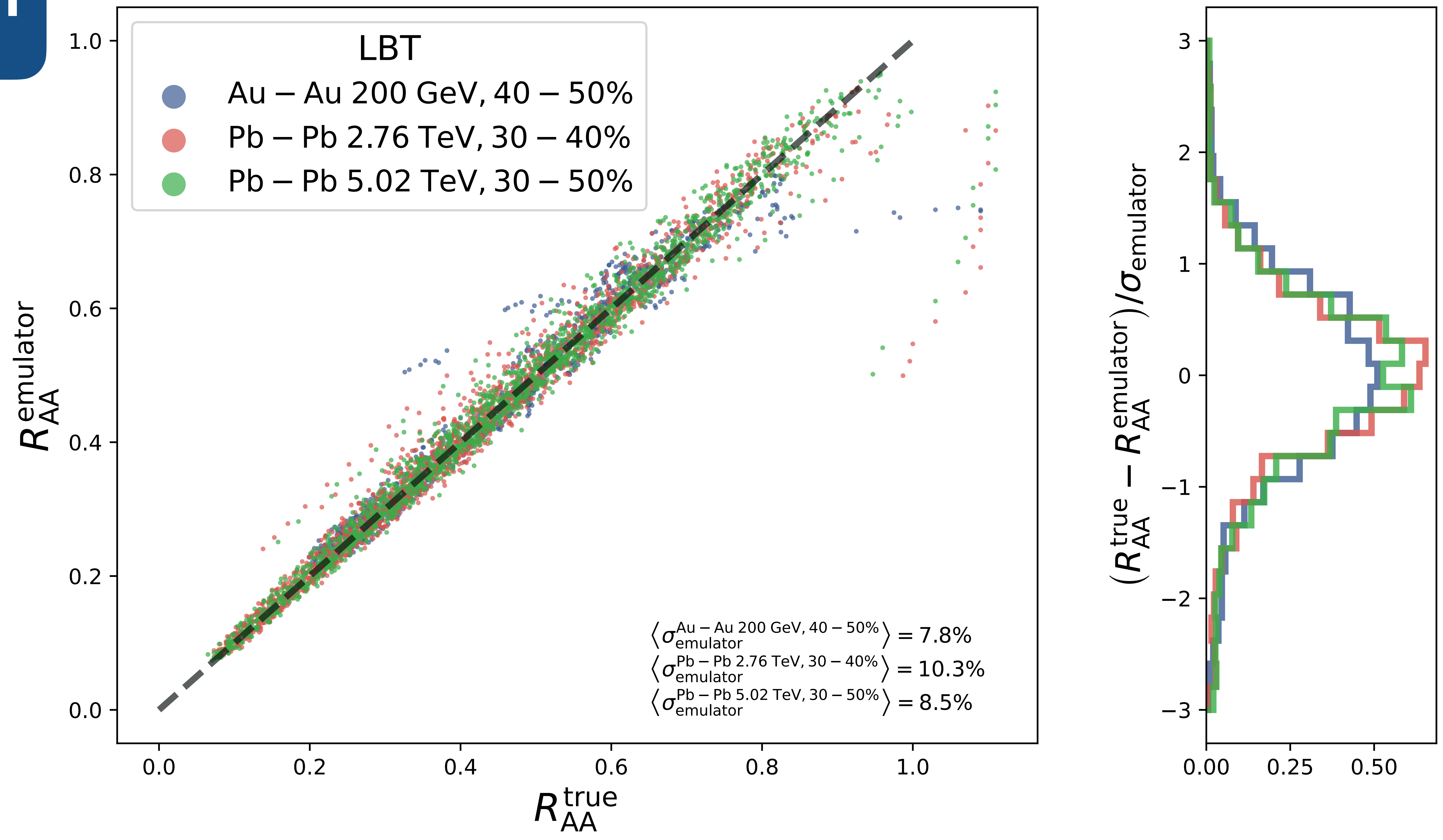

Results

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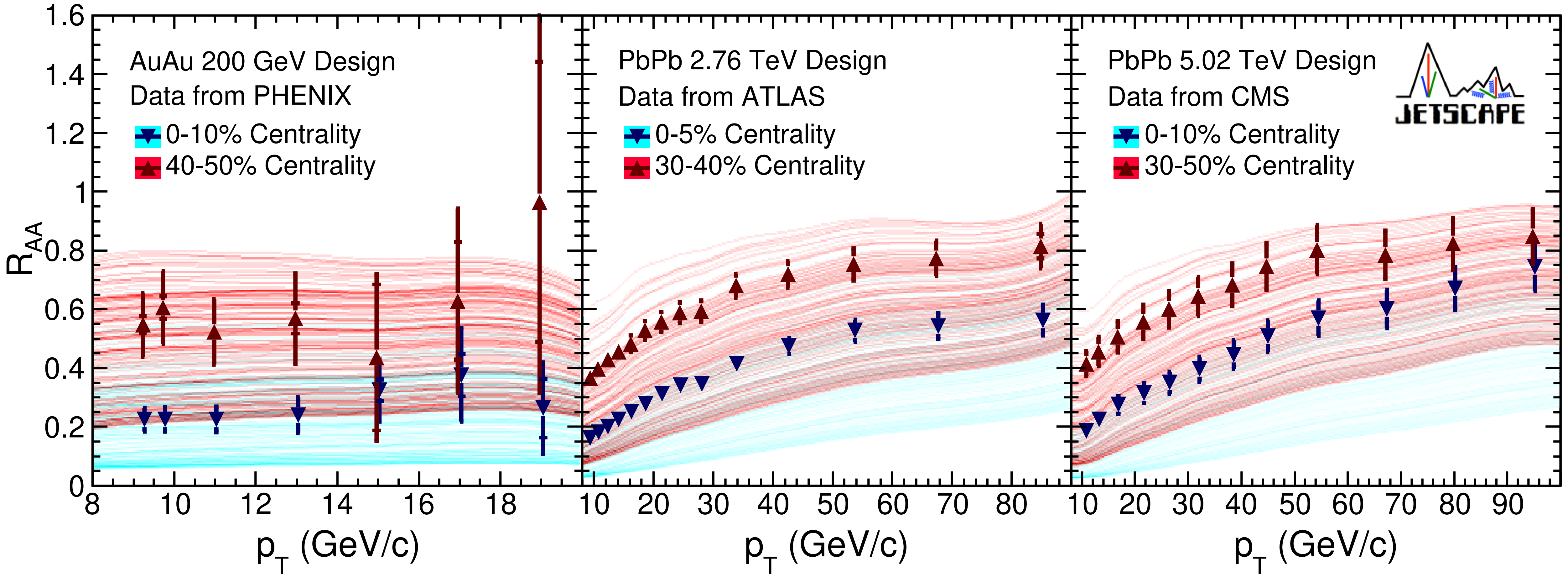
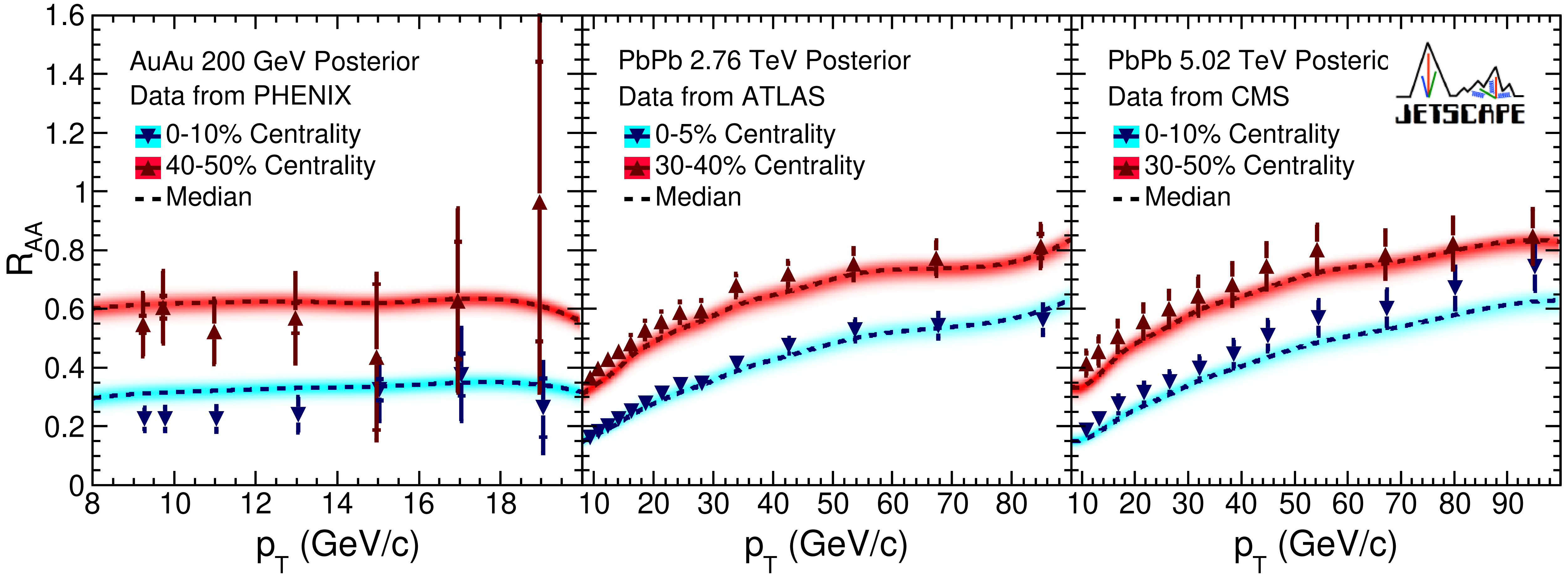
LBT model



Prior

↓

Posterior



LBT describes the data reasonably well

Some small systematic deviations

Results

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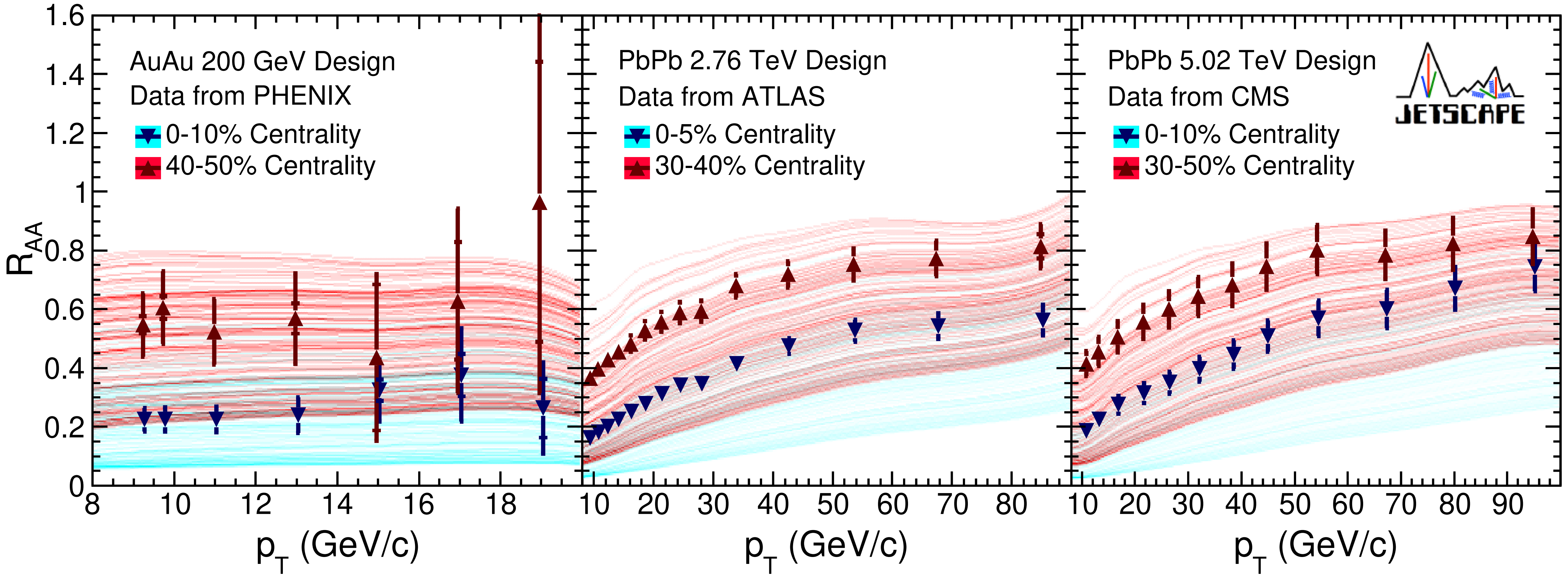

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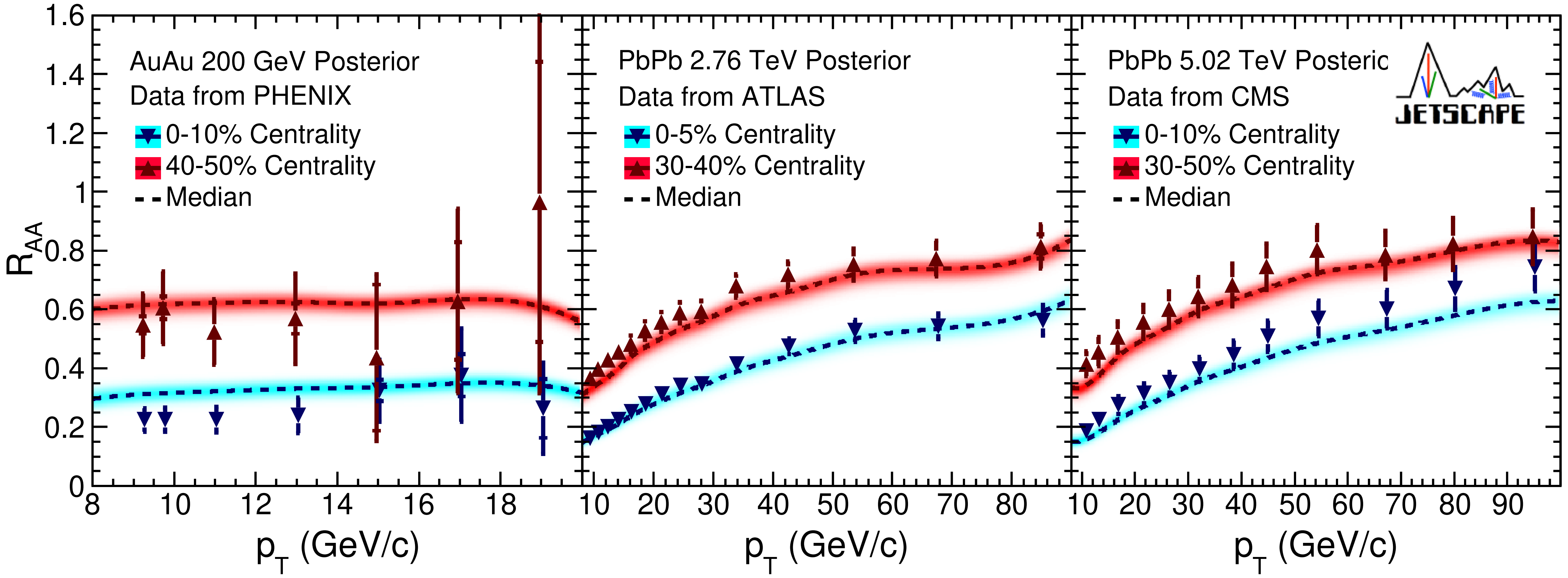


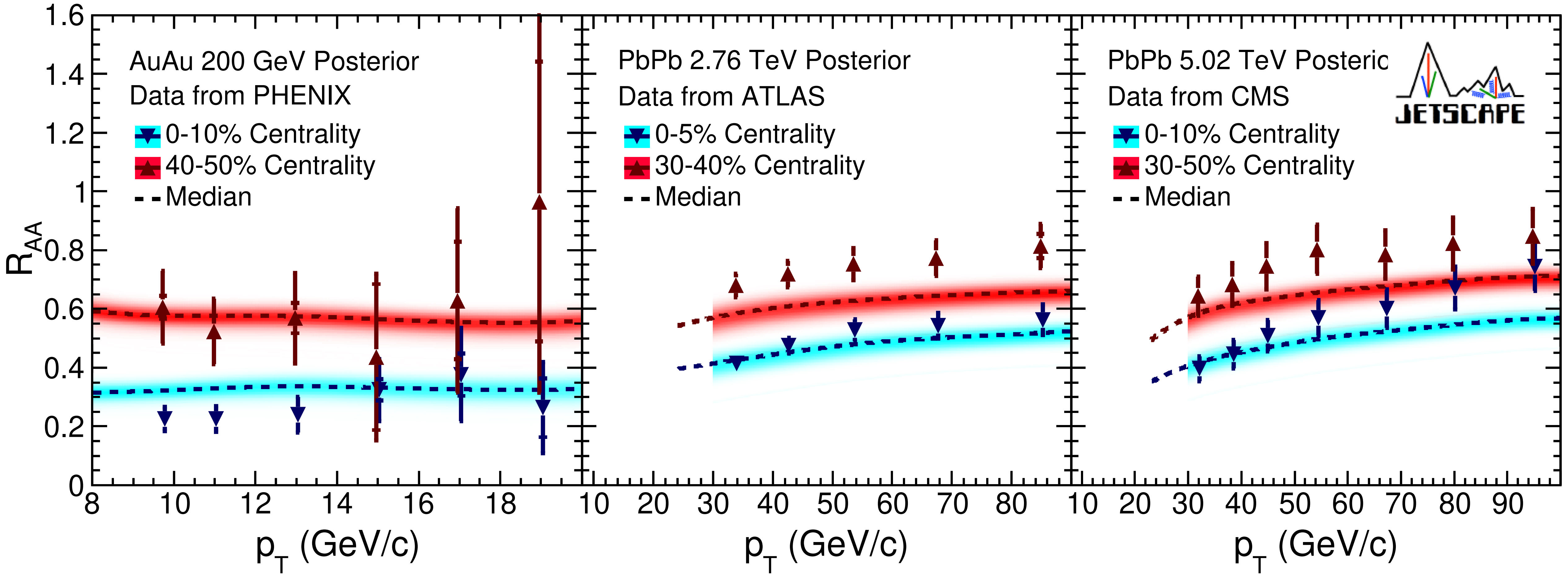
LBT describes the data reasonably well

Some small systematic deviations

MATTER describes the data slightly less well

MATTER expected to be valid only at sufficiently high p_{T}; fit restricted

MATTER model



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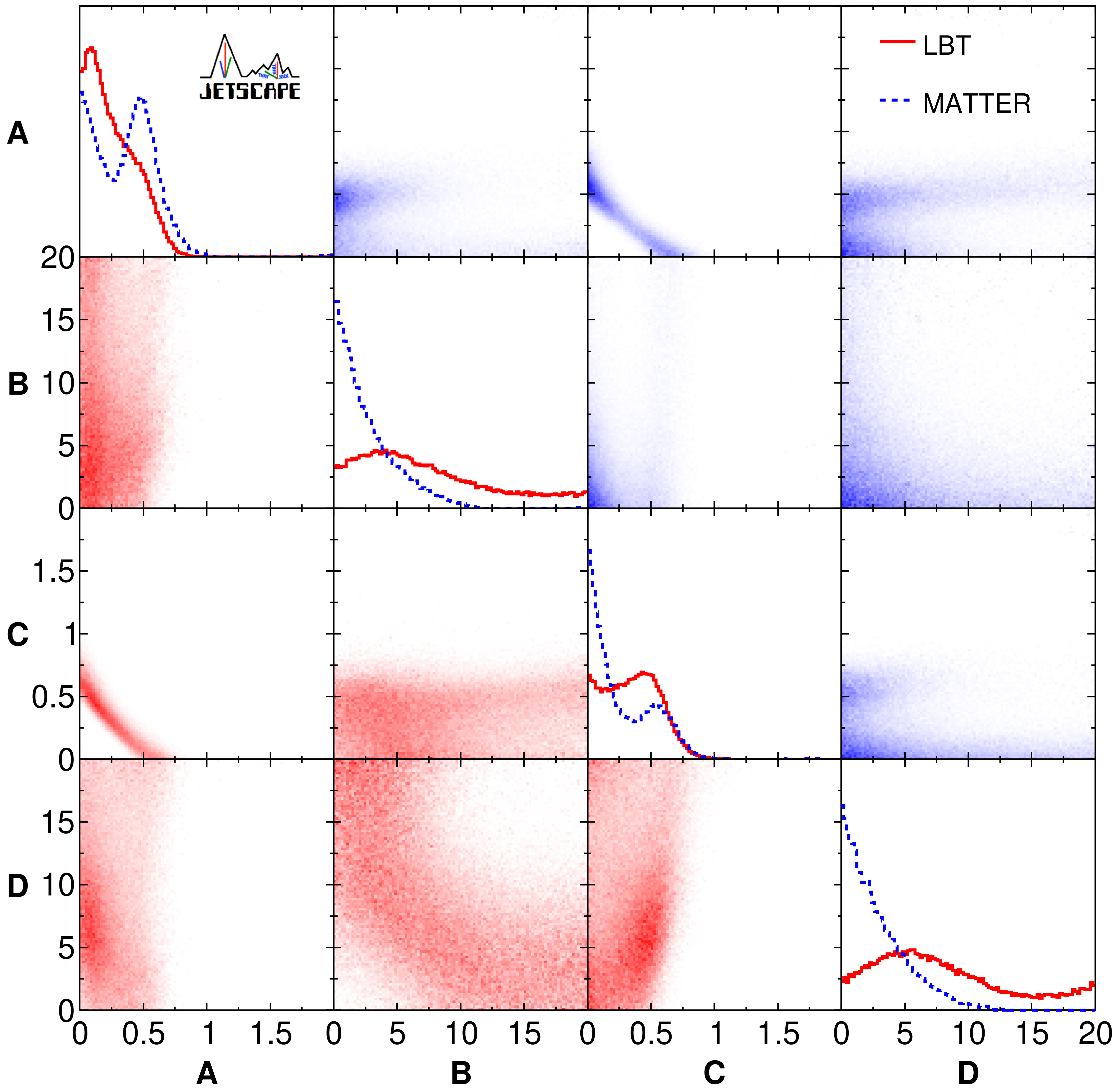

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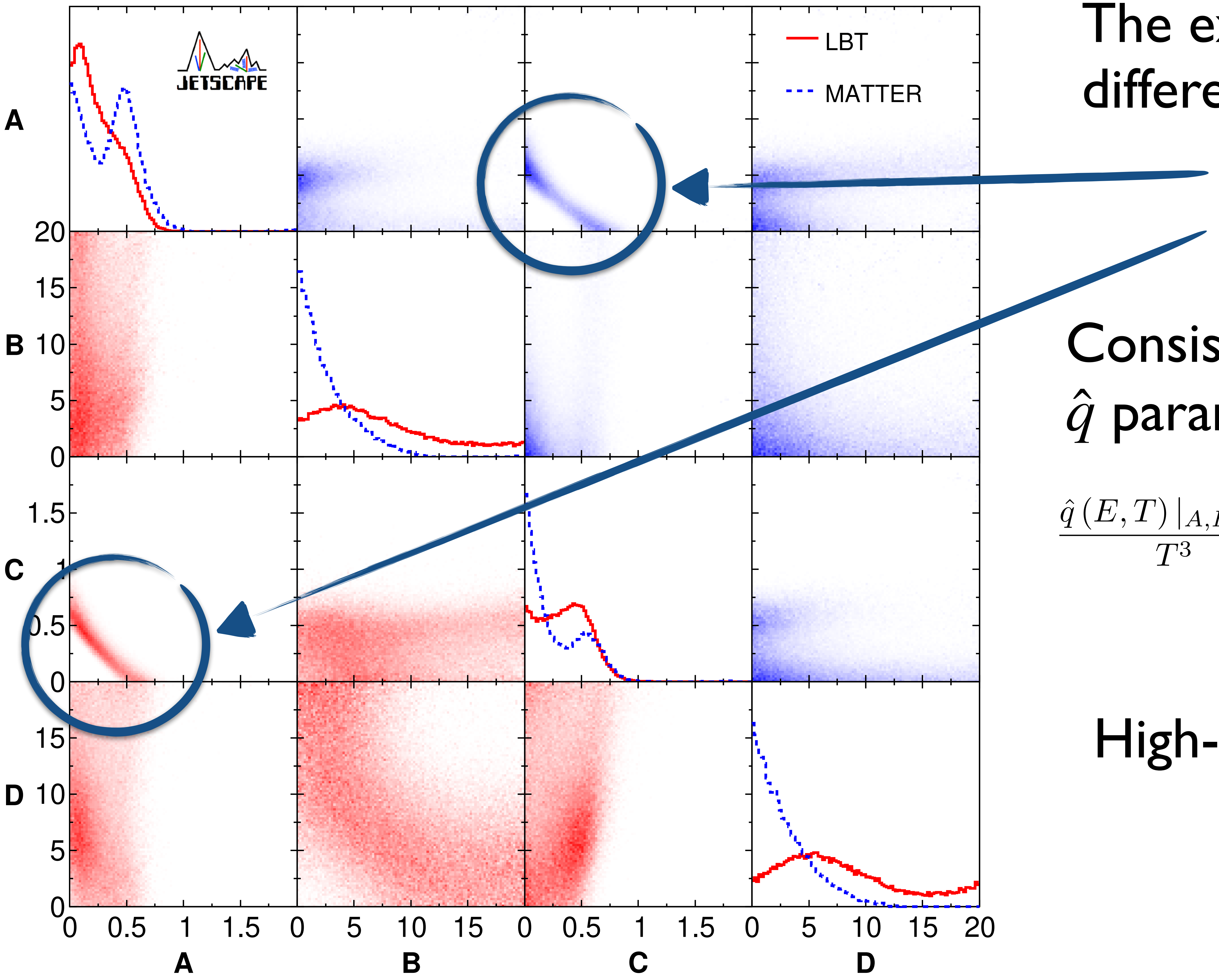

The extracted parameters are substantially different for MATTER vs. LBT

MATTER: large A, small C

LBT: small A, large C

Consistent with the original motivation of the $\hat{q}$ parameterization:

$$\frac{\hat{q}\left(E,T\right)|_{A,B,C,D}}{T^3} = 42C_R\frac{\zeta(3)}{\pi}\left(\frac{4\pi}{9}\right)^2\left\{\frac{A\left[\ln\left(\frac{E}{\Lambda}\right)-\ln(B)\right]}{\left[\ln\left(\frac{E}{\Lambda}\right)\right]^2}+\frac{C\left[\ln\left(\frac{E}{T}\right)-\ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2}\right\}$$

High-virtuality inspired

T-independent

HTL-inspired

elastic scattering off temperature T

Results

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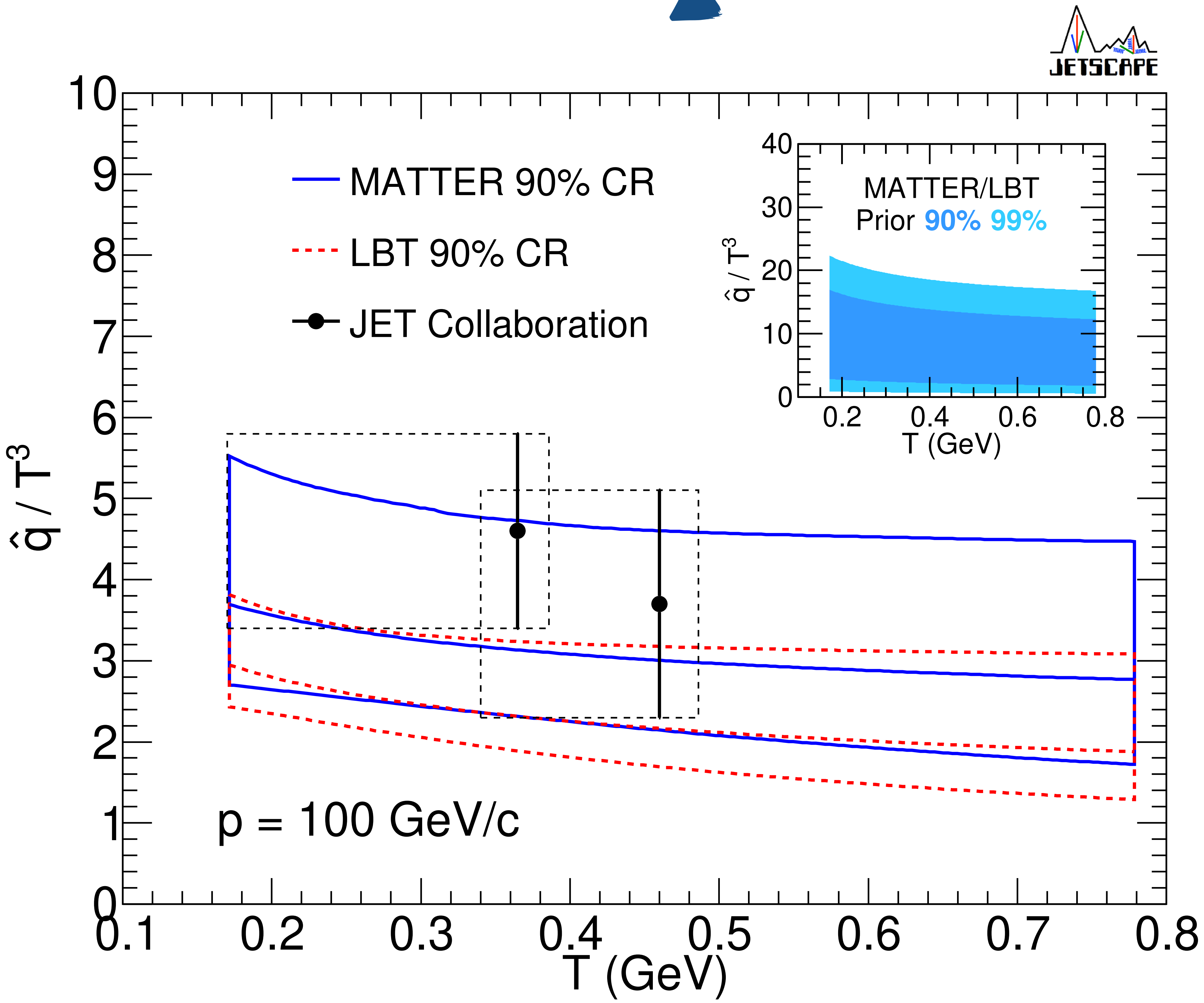
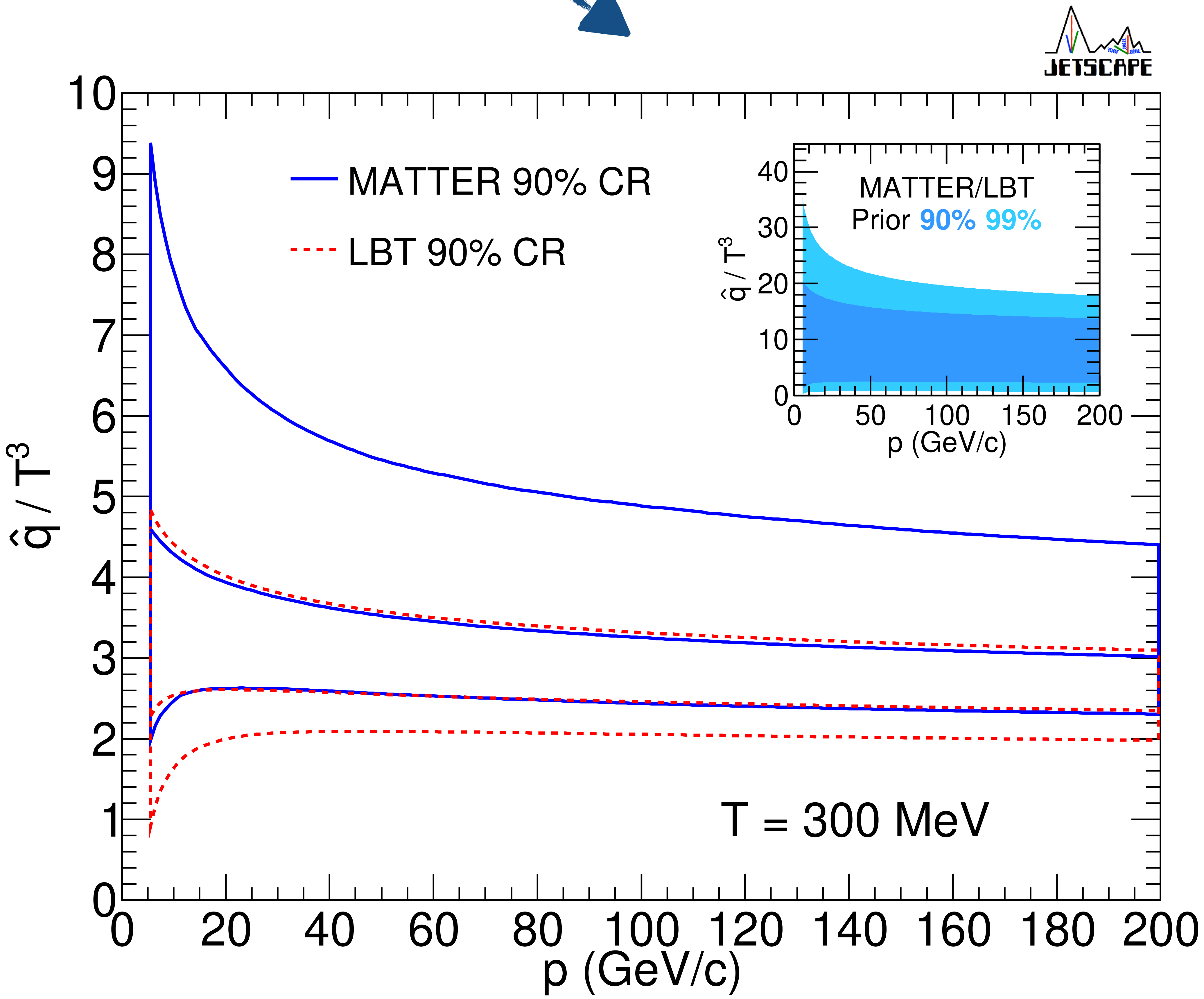
From these extracted parameters, we plot the extracted $\hat{q}$

Weak dependence on T, p





Consistent T-dependence with JET Collaboration

Smaller median: elastic scattering, multiple gluon emission

(Plotted $\hat{q}$ is for quarks)

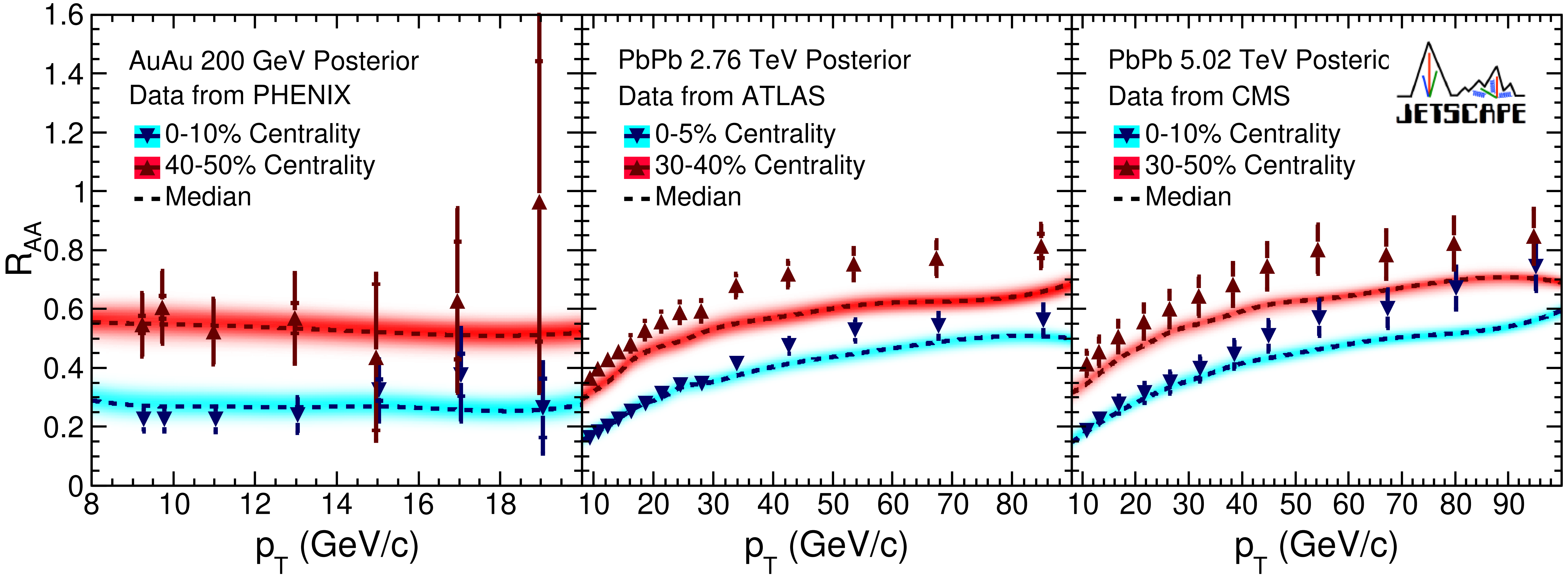
Multi-stage model

2102.11337

Theoretical arguments suggest that a multi-stage model is more well-founded:

MATTER — high-virtuality, $Q > Q_0$
LBT — low-virtuality, $Q < Q_0$

→ Include additional parameter, Q_0, to the fit



No evidence that multi-stage model improves agreement with data

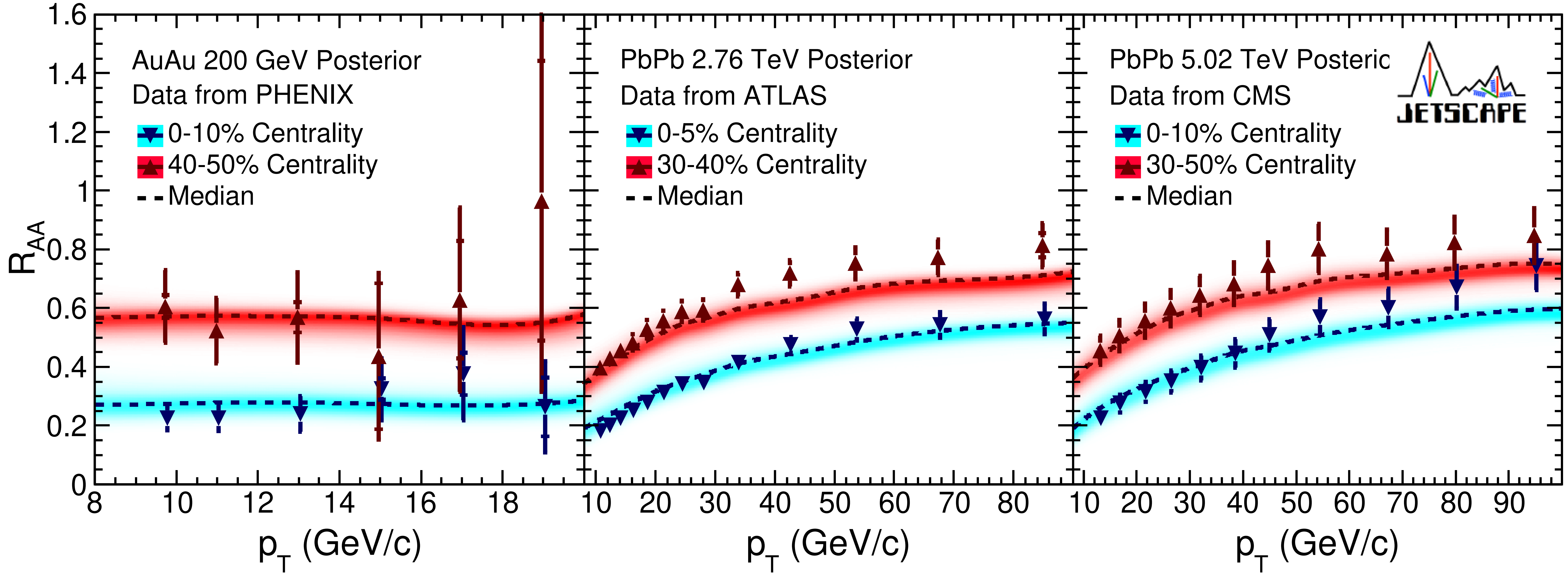
- ☐ Caveat: p_T range not restricted as in MATTER only case

Multi-stage model

2102.11337

We also explored an alternate multi-stage parameterization, in which we replace the “high-virtuality" term with $E \rightarrow Q$

$$\frac{\hat{q}\left(Q,E,T\right)|_{Q_0,A,C,D}}{T^3} = 42C_R\frac{\zeta(3)}{\pi}\left(\frac{4\pi}{9}\right)^2\left\{\frac{A\left[\ln\left(\frac{Q}{\Lambda}\right)-\ln\left(\frac{Q_0}{\Lambda}\right)\right]}{\left[\ln\left(\frac{Q}{\Lambda}\right)\right]^2}\theta(Q-Q_0)+\frac{C\left[\ln\left(\frac{E}{T}\right)-\ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2}\right\}$$



Improved fit

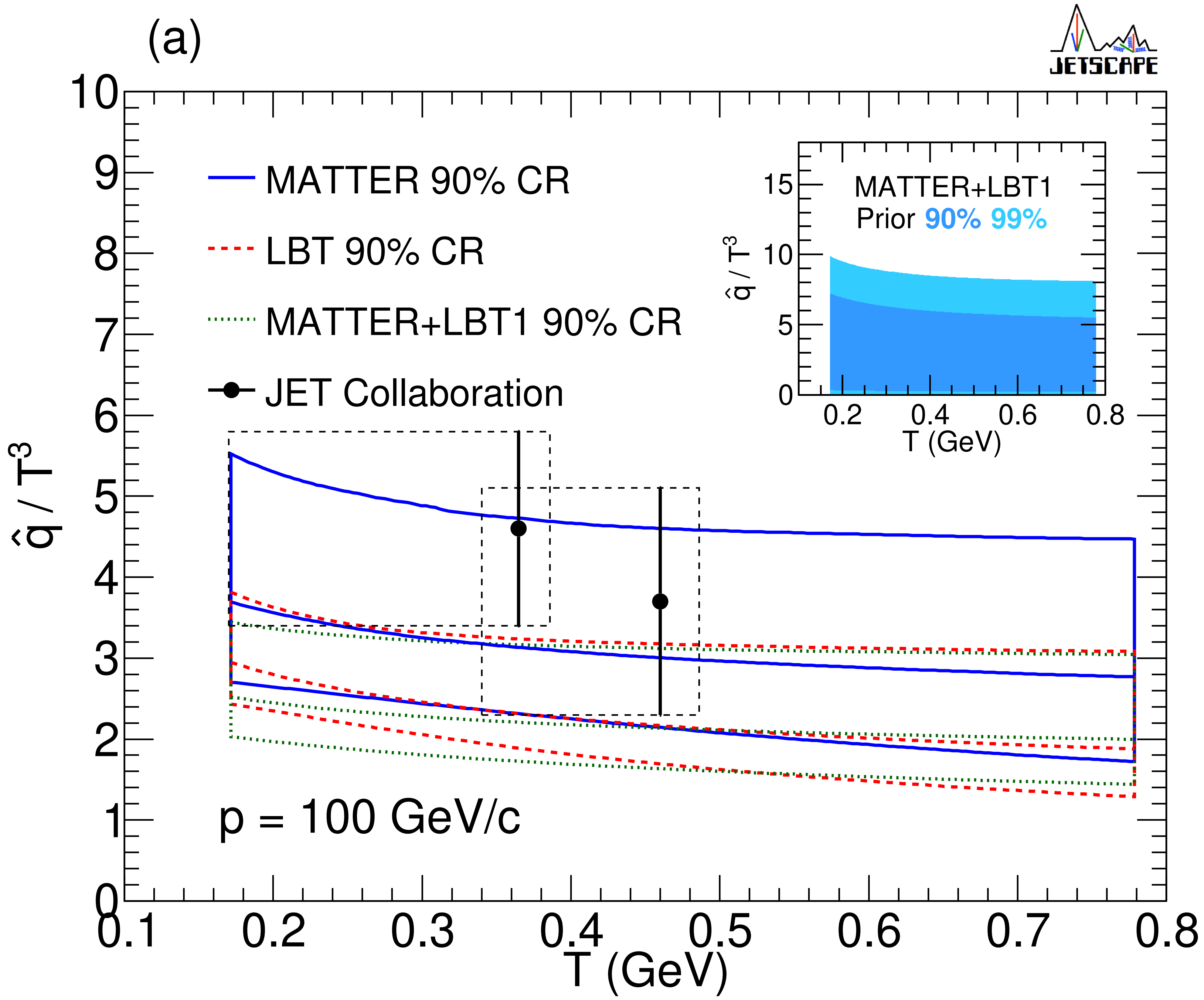
Will require additional observables to make more definitive statement about multi-stage model

Multi-stage model

2102.11337

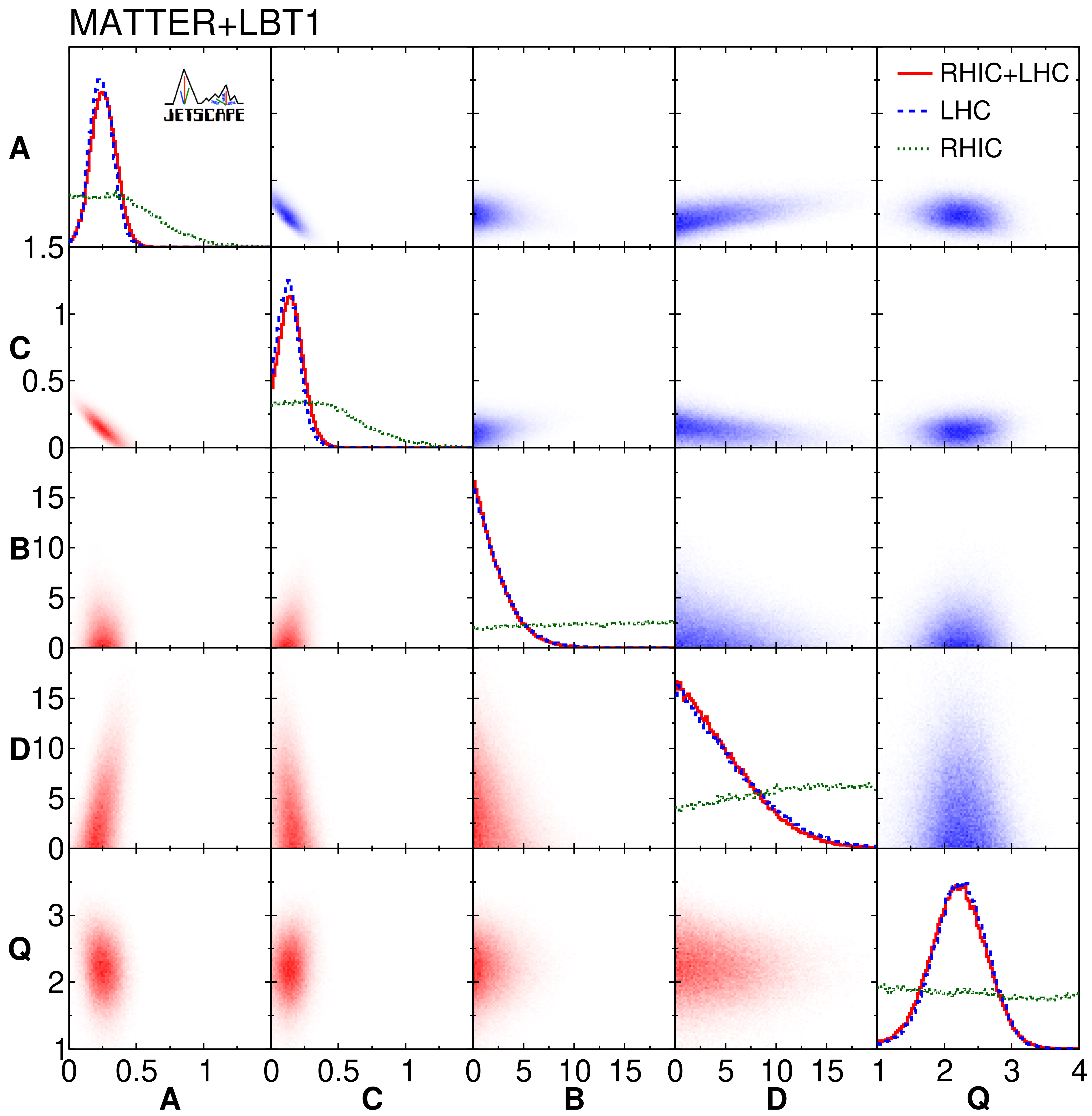
Extracted $\hat{q}$ of MATTER+LBT is smaller than MATTER,LBT alone

Due to additional quenching at low virtuality (compared to MATTER) or high virtuality (compared to LBT alone)



Multi-stage model

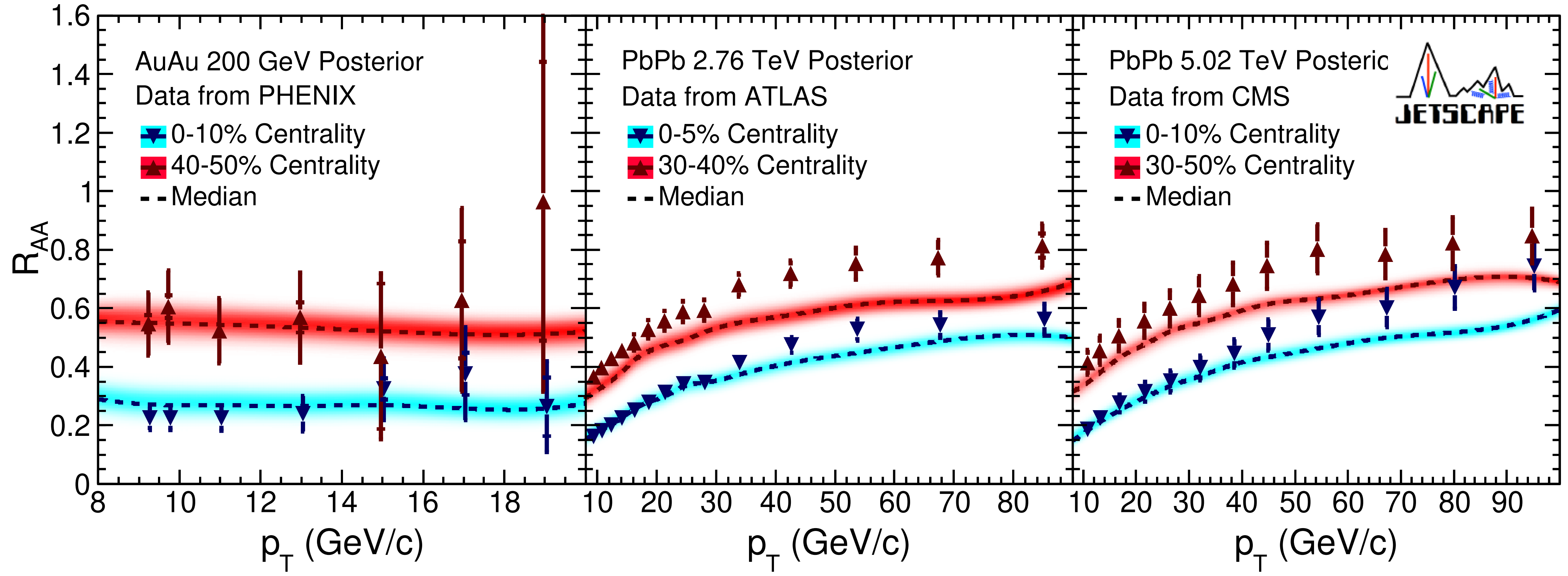
2102.11337



We also test the impact of RHIC vs. LHC data

Fit dominated by LHC data

Due to choice of cutoff: $p_{\mathrm{T}} < 8 \ \mathrm{GeV}/c$



First extraction of virtuality-switching parameter: $Q_0 \sim 2 - 3 \ \mathrm{GeV}/c$

Summary

We extracted $\hat{q}(E,T)$ as a continuous function of E, T using Bayesian parameter estimation with inclusive hadron R_{AA} data

- ☐ Several JETSCAPE models considered: MATTER, LBT, MATTER+LBT
- ☐ Data significantly constrains prior distributions
- ☐ No evidence for multi-stage model being preferred by data

Summary

We extracted $\hat{q}(E,T)$ as a continuous function of E,T using Bayesian parameter estimation with inclusive hadron $R_{\rm AA}$ data

- ☐ Several JETSCAPE models considered: MATTER, LBT, MATTER+LBT
- ☐ Data significantly constrains prior distributions
- ☐ No evidence for multi-stage model being preferred by data

Global analysis is key to the future of the jet quenching physics program

- ☐ Extension to additional observables — jet $R_{\rm AA}$, substructure, correlations
  - ☐ **Need theory input:** model parameterizations, multi-stage paradigm, improved modeling of heavy-ion stages (hydro calibration, quenching in hadronic phase), …
  - ☐ **Need experiment input:** reporting of uncertainty correlations on HEPData
- ☐ Provide experimental guidance — observables, systems, centrality to best constrain models