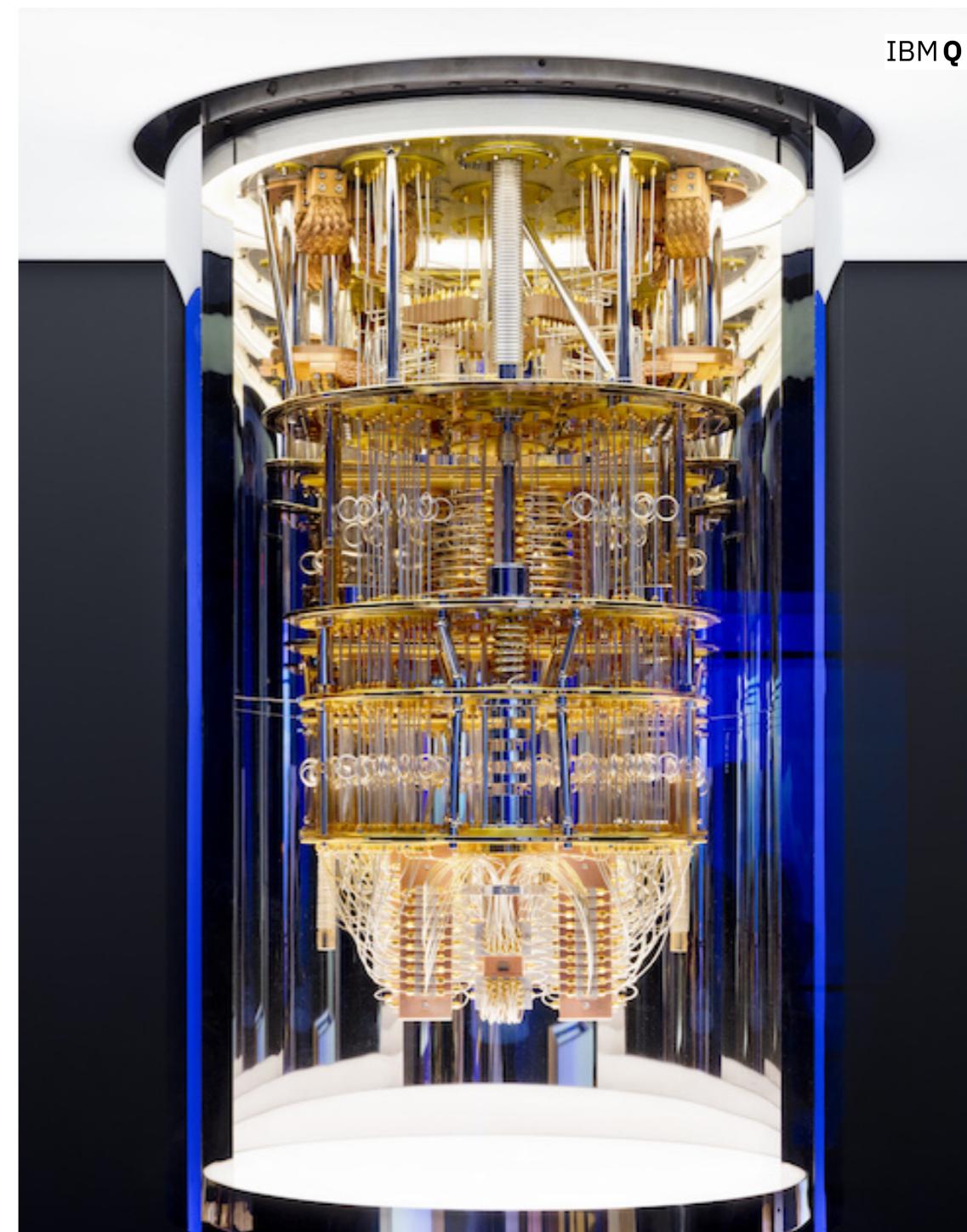


Simulating real-time dynamics of hard probes in nuclear matter on a quantum computer

James Mulligan
Lawrence Berkeley National Laboratory

arXiv: 2010.03571

Wibe de Jong	
Mekena Metcalf	↳ LBNL (Quantum Information)
James Mulligan	
Mateusz Ploskon	↳ LBNL (Nuclear Science)
Felix Ringer	
Xiaojun Yao	↳ MIT (Nuclear Theory)



Particle-Astro-Nuclear Seminar
Wayne State University
Nov 6 2020

Outline

Hard probes in the
quark-gluon plasma

Open quantum systems in
heavy-ion collisions

Quantum simulation
with IBM Q

Outline

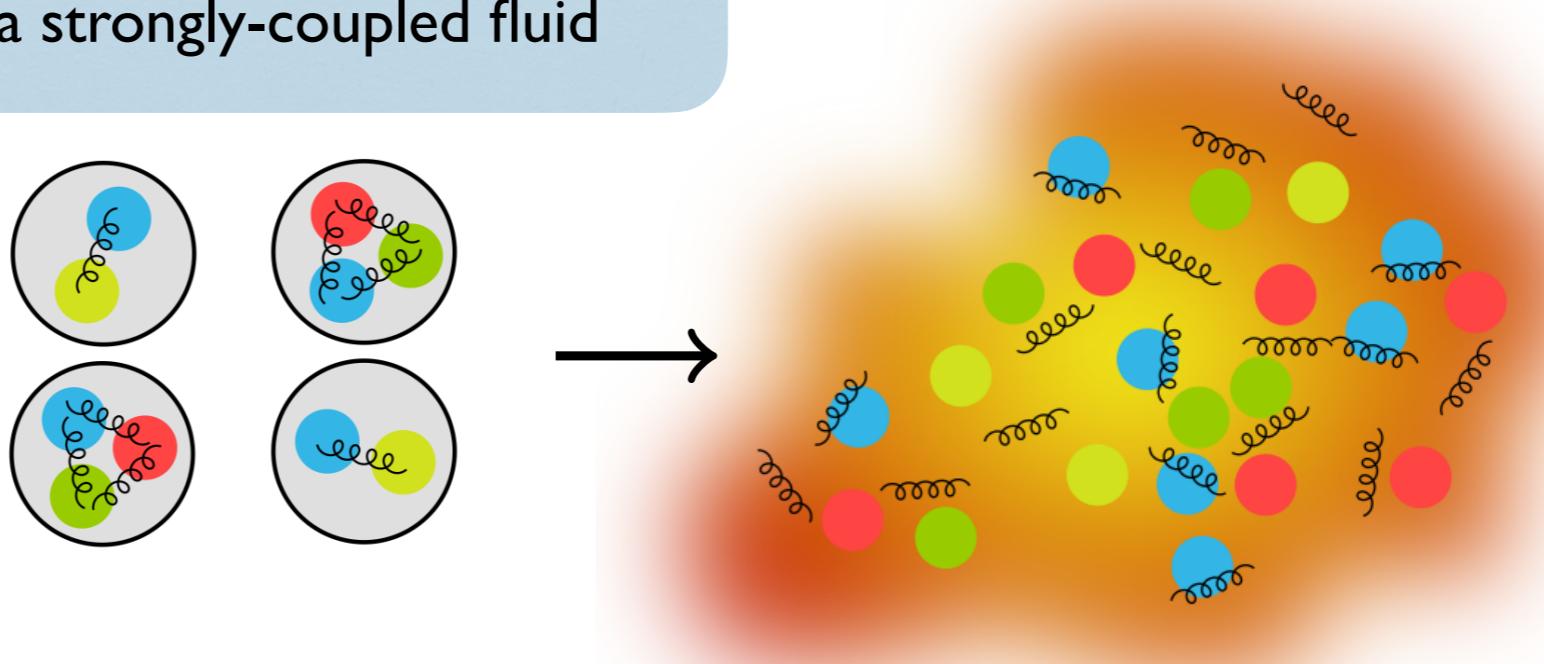
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The quark-gluon plasma

If we heat nuclear matter to $T = \mathcal{O}(100 \text{ MeV})$, quarks and gluons become **deconfined** into a strongly-coupled fluid



The quark-gluon plasma is a laboratory to understand how complex properties arise from Quantum Chromodynamics

How does this strongly-coupled fluid emerge from the Lagrangian?

Does deconfined QCD have quasi-particle structure?

How does color confinement emerge?

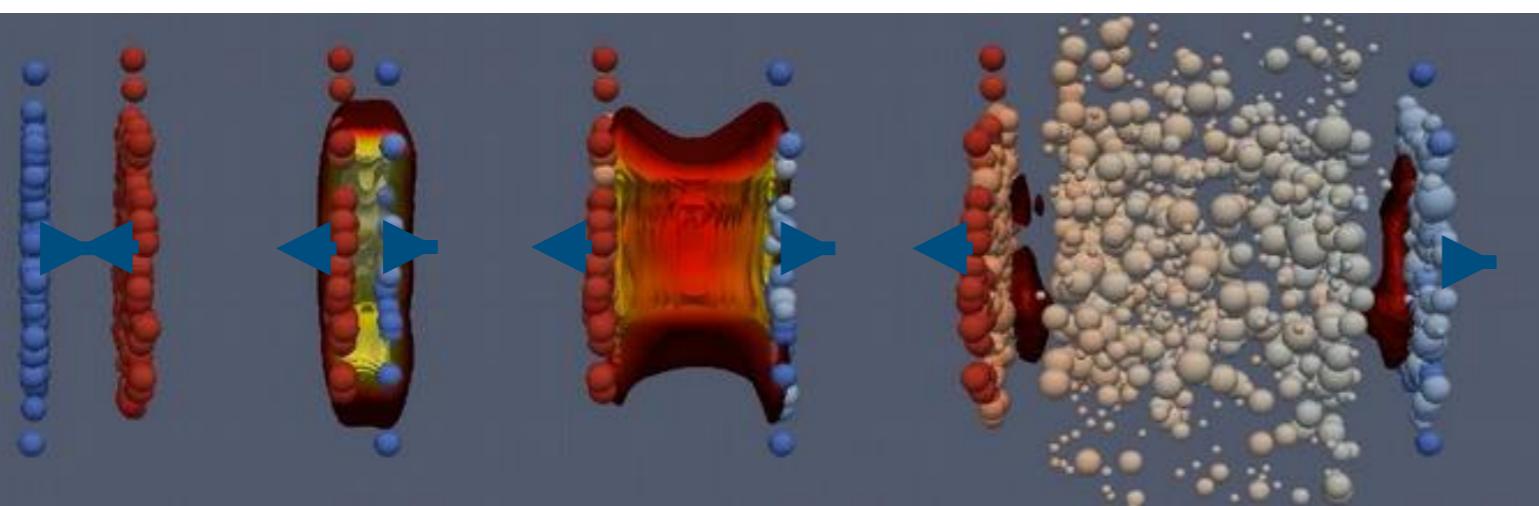
Heavy-ion collisions



We collide nuclei at

Large Hadron Collider (LHC)
Relativistic Heavy Ion Collider (RHIC)

to produce a hot, dense state of matter known as the quark-gluon plasma



Soft collisions transform kinetic energy of nuclei into region of **large energy density** for $t \sim \mathcal{O}(10 \text{ fm}/c)$

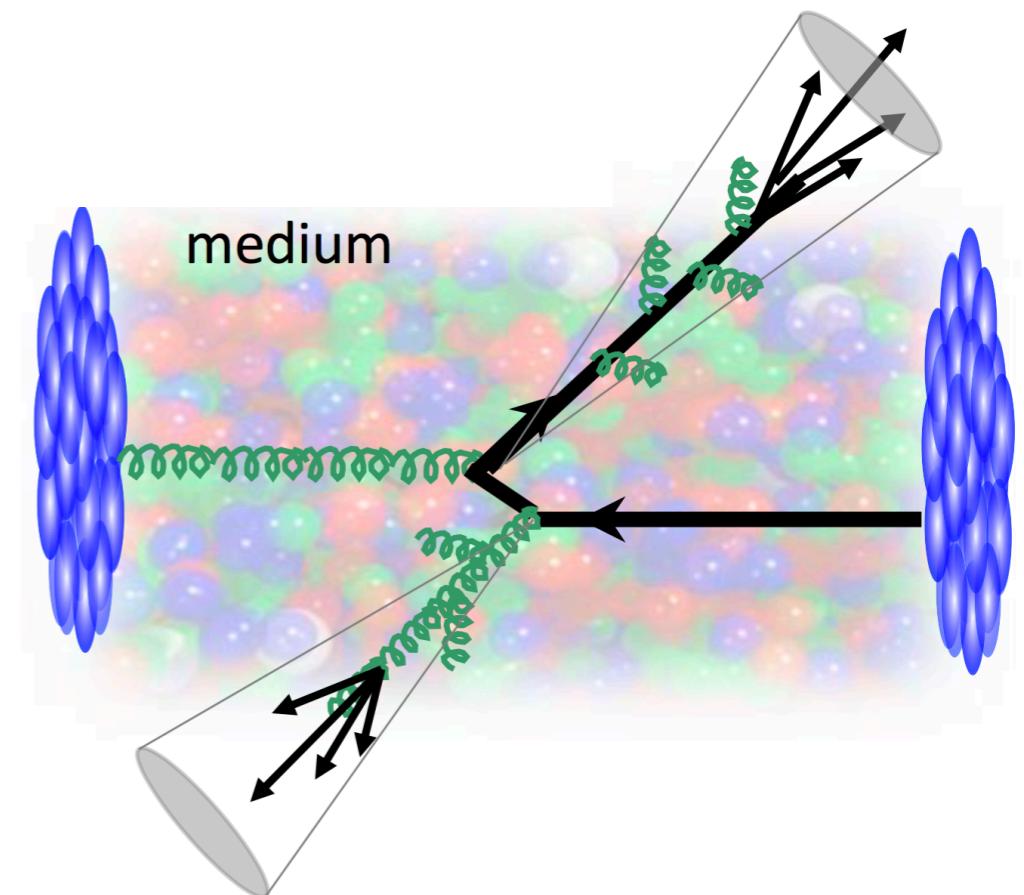
Hard probes of the quark gluon plasma

In addition to soft scatterings, there are occasional hard scatterings ($\text{large-}Q^2$) in the collisions

- Highly energetic particles: jets
- Large mass particles: heavy quarks

These “hard probes” interact with the quark-gluon plasma as they traverse it

- By modeling these interactions, we hope to determine the structure of the QGP



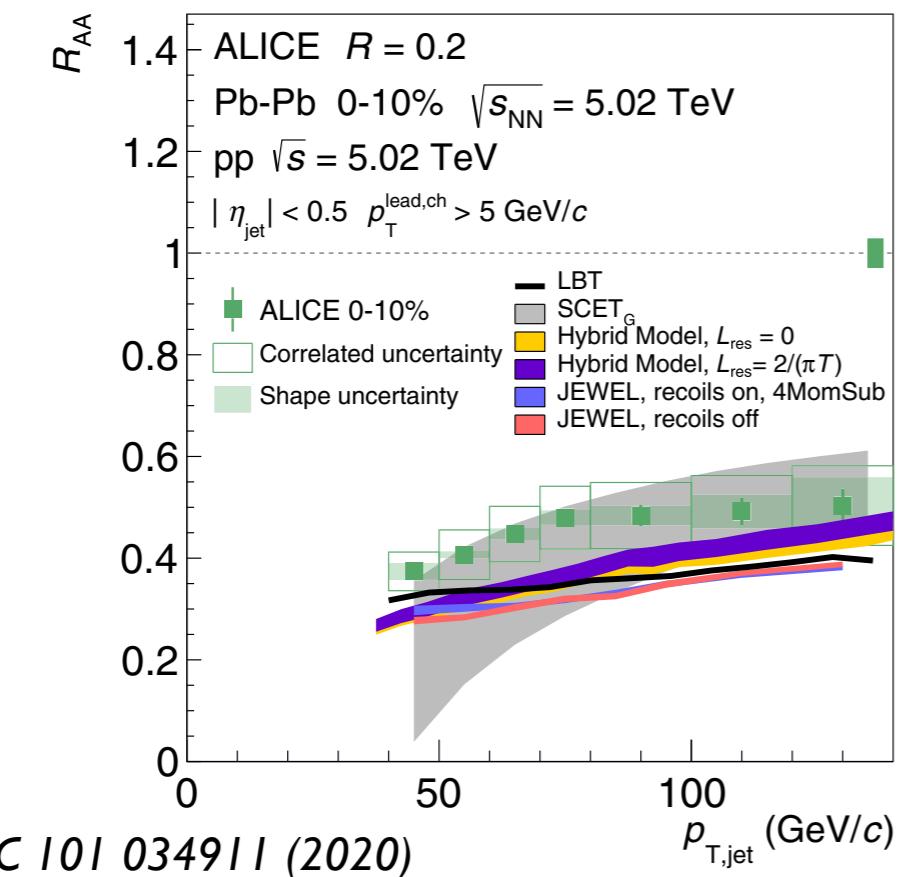
Hard probes — experiment

Experiments measure how cross-sections of hard probes are modified in heavy-ion collisions compared to proton-proton collisions

$$R_{AA} = \frac{d\sigma^{\text{PbPb}}}{\langle T_{AA} \rangle d\sigma^{pp}}$$

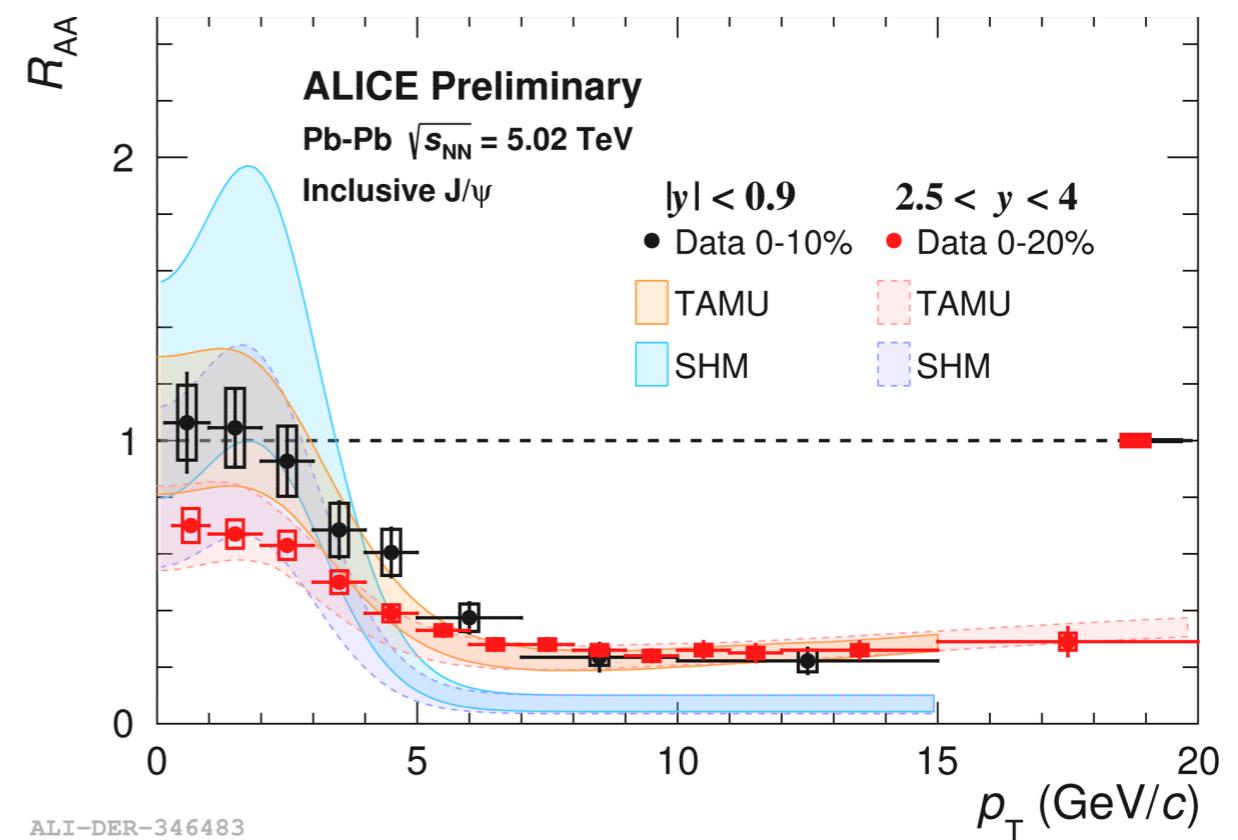
Jets

Jet yields are suppressed due to “energy loss” to the dense medium



Heavy quarks

Heavy quark bound pairs (quarkonium) are “melted” by the hot medium

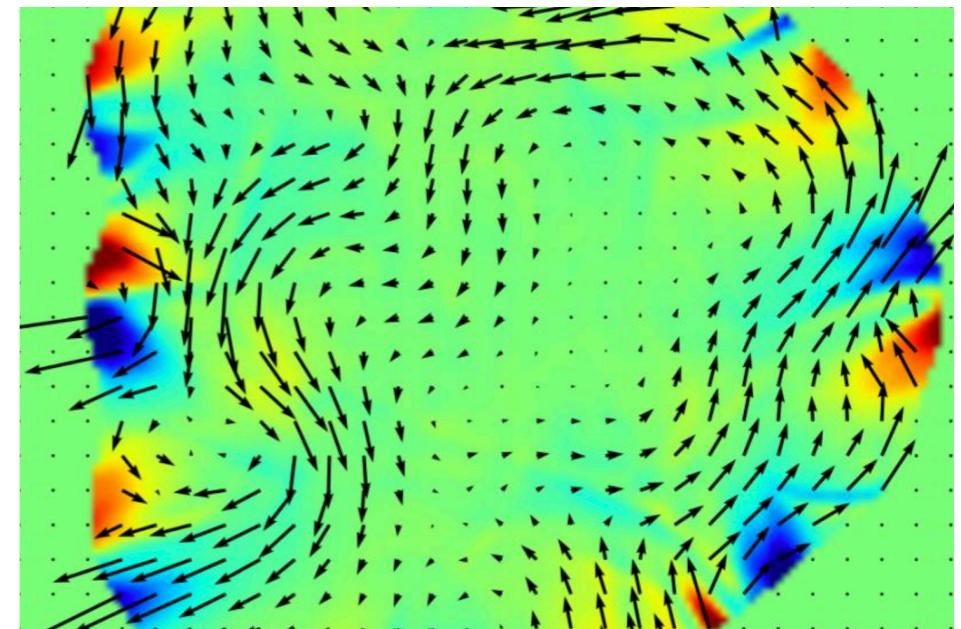


Hard probes — theory

In vacuum: calculate scattering of asymptotic states using perturbative QCD

Note that there is no sense of “time evolution”

In medium: must combine probe evolution with hydrodynamic evolution of the QGP



X.N. Wang

In heavy-ion collisions, the modifications of the probe due to its evolution through the QGP are typically put in “by hand”, rather than a true real-time evolution

Medium-modified parton shower

see e.g. Majumder PRC 88 (2013)

Real-time evolution

Is there a way we can compute real-time evolution in QCD?

One could think to use **lattice QCD** to numerically solve for the modification of the probe

see e.g. Kumar, Majumder, Weber 2010.14463 (2020)

However, lattice QCD encounters a **sign problem**

$$\int e^{i\mathcal{L}t}$$

which is typically alleviated by solving in Euclidean time $t \rightarrow it$
(i.e. not real-time evolution)

But **quantum computing** offers a way to do real-time evolution!

→ Hamiltonian formulation of QCD

see e.g. Preskill '18

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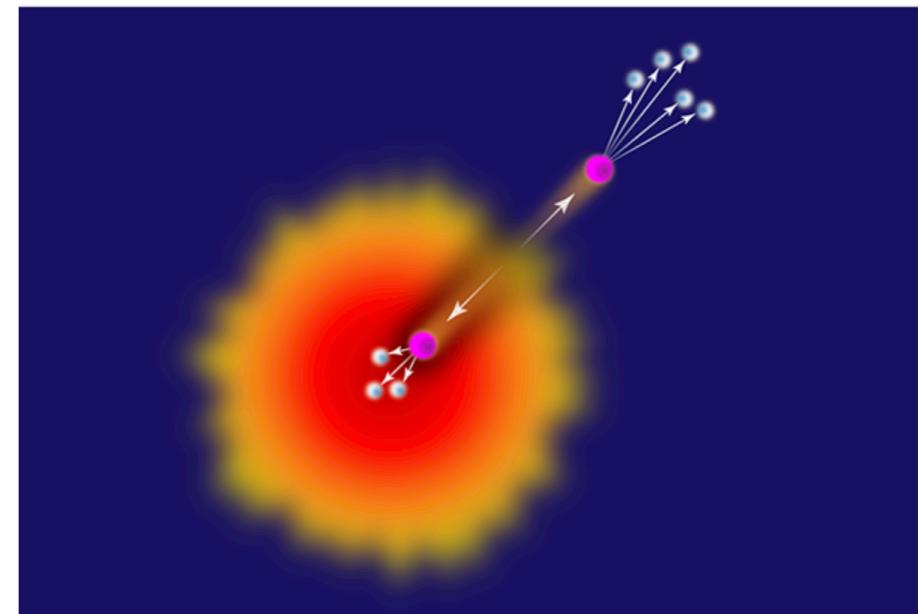
Open quantum systems and the nuclear medium

Study the real time dynamics of the quantum evolution of probes in the nuclear medium (LHC/RHIC/EIC)

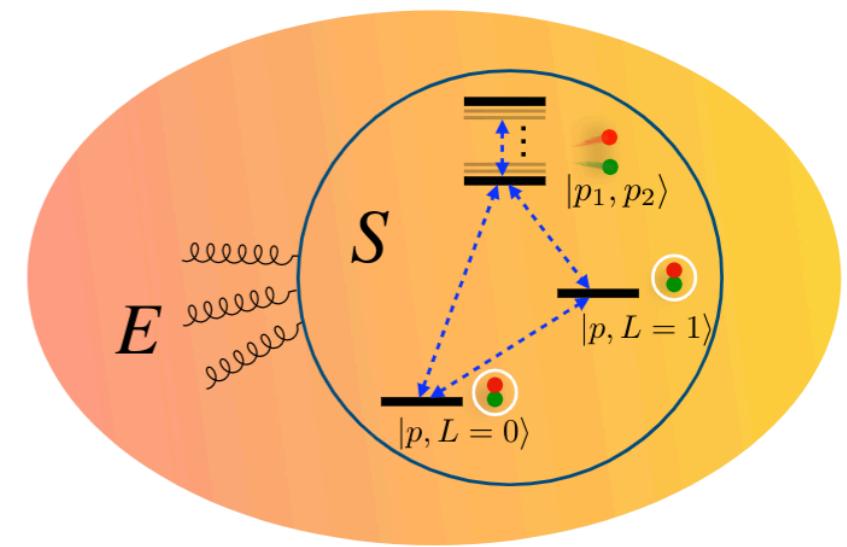
In principle one could solve for the entire QGP evolution
But here we focus on simulating the probe

System - Jet/heavy-flavor

Environment - Nuclear matter



$$H(t) = H_S(t) + H_E(t) + H_I(t)$$



Akamatsu, Rothkopf '12-'20, Müller et al '18, Mehen, Yao '18,
Qiu, Ringer, Sato, Zurita '19, Vaidya, Yao '20

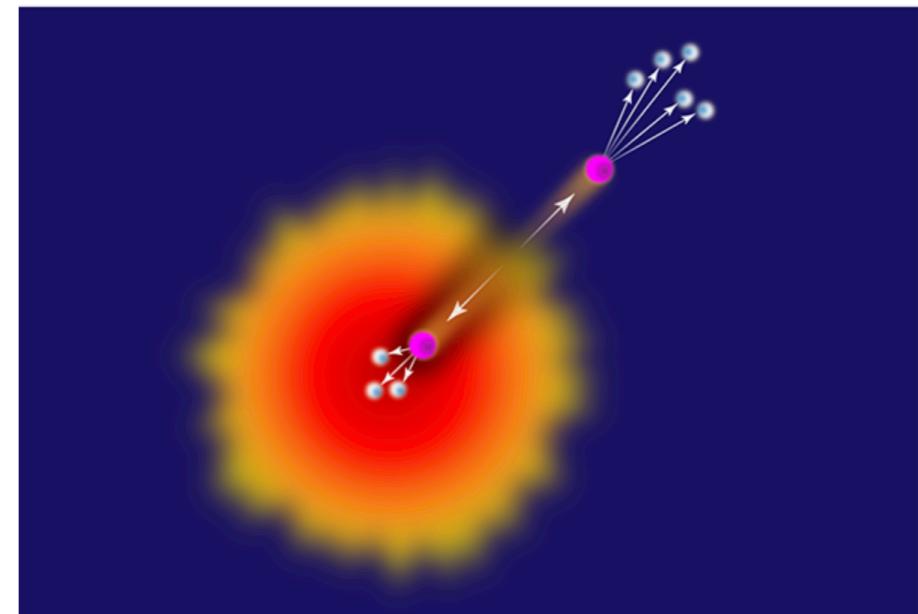
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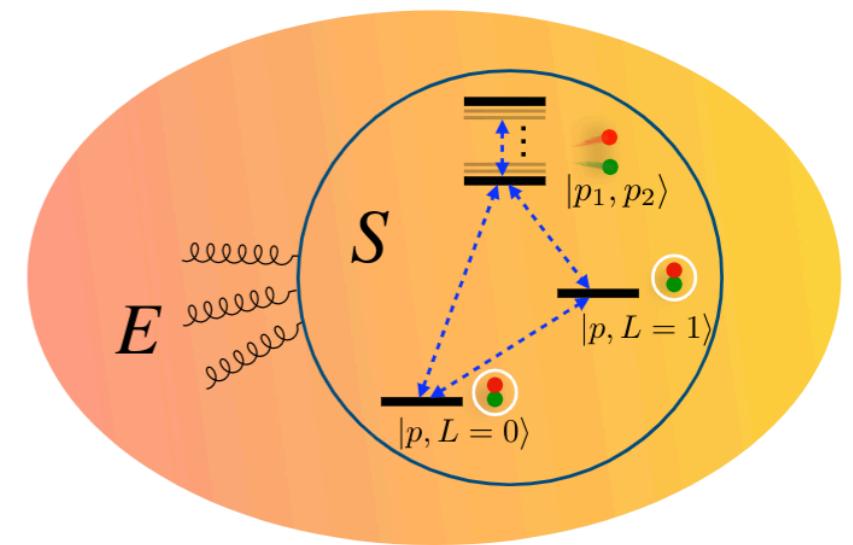
Environment - Nuclear matter



$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

The time evolution is governed by the von Neumann equation:

$$\frac{d}{dt} \rho^{(\text{int})}(t) = -i [H_I^{(\text{int})}(t), \rho^{(\text{int})}(t)]$$

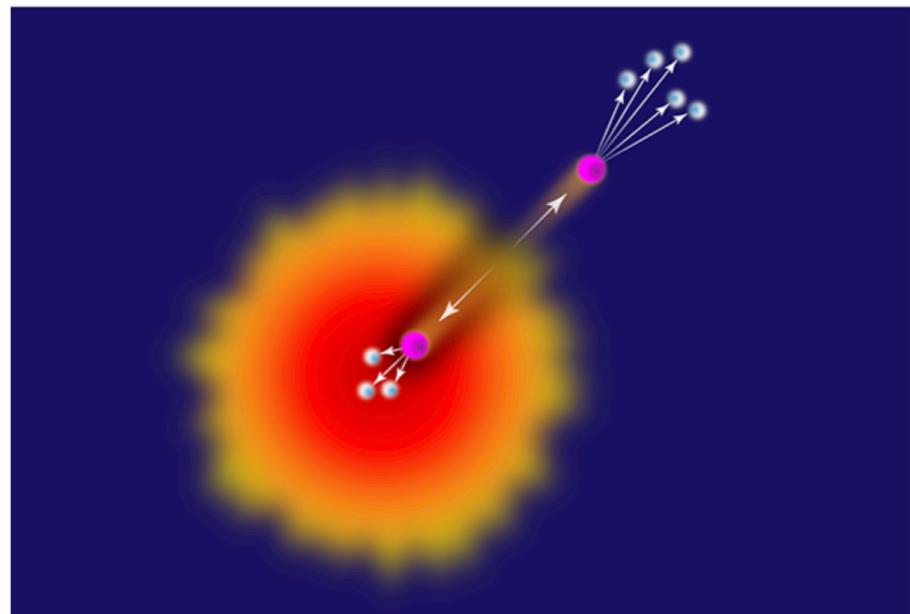


Akamatsu, Rothkopf '12-'20, Müller et al '18, Mehen, Yao '18,
Qiu, Ringer, Sato, Zurita '19, Vaidya, Yao '20

Open quantum systems and the nuclear medium

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System - Jet/heavy-flavor

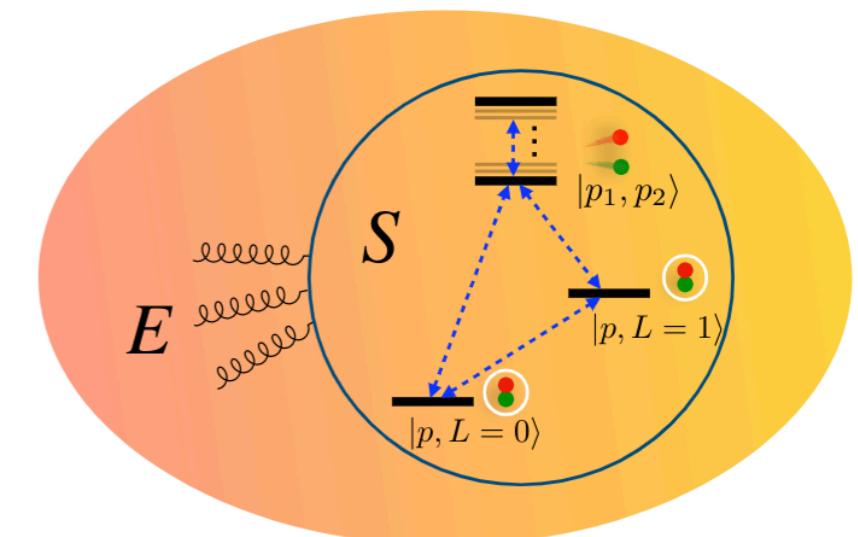
Environment - Nuclear matter

$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

In the Markovian limit, the subsystem is described by
a **Lindblad equation**

$$\frac{d}{dt}\rho_S = -i[H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right)$$

$$\rho_S = \text{tr}_E[\rho]$$

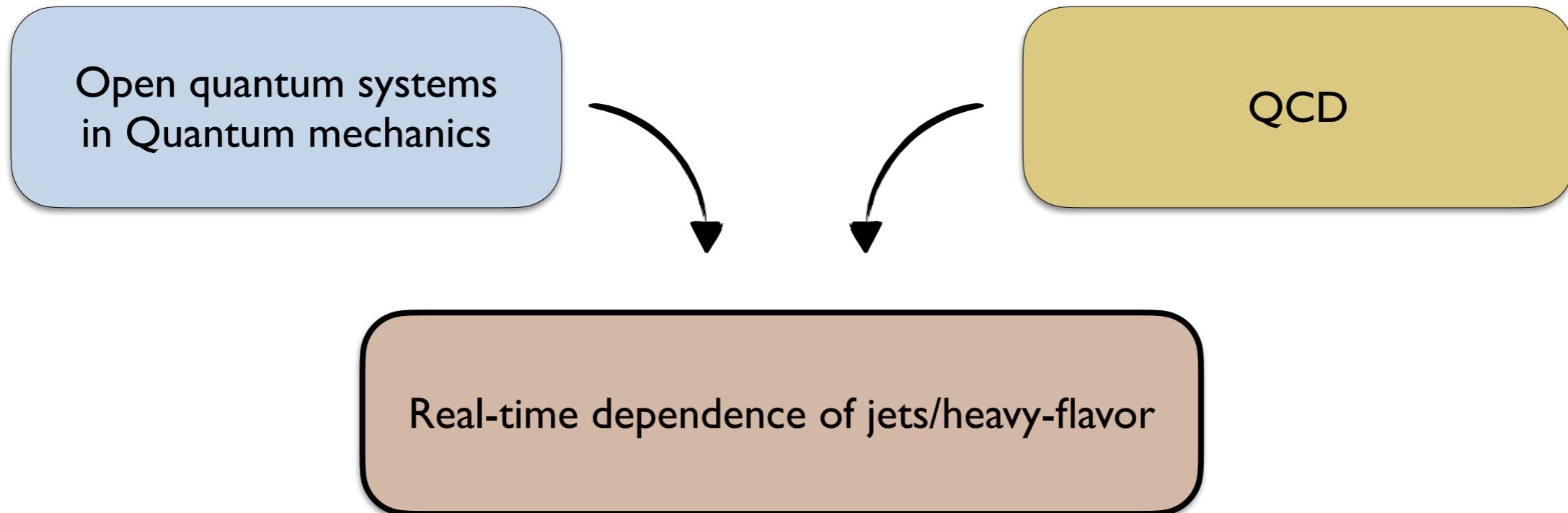


See also e.g. non-global logs and CGC

Neill '15, Armesto et al. '19, Li, Kovner '20

Akamatsu, Rothkopf '12-'20, Müller et al '18, Mehen, Yao '18, Qiu, Ringer, Sato, Zurita '19, Vaidya, Yao '20

Open quantum systems and the nuclear medium



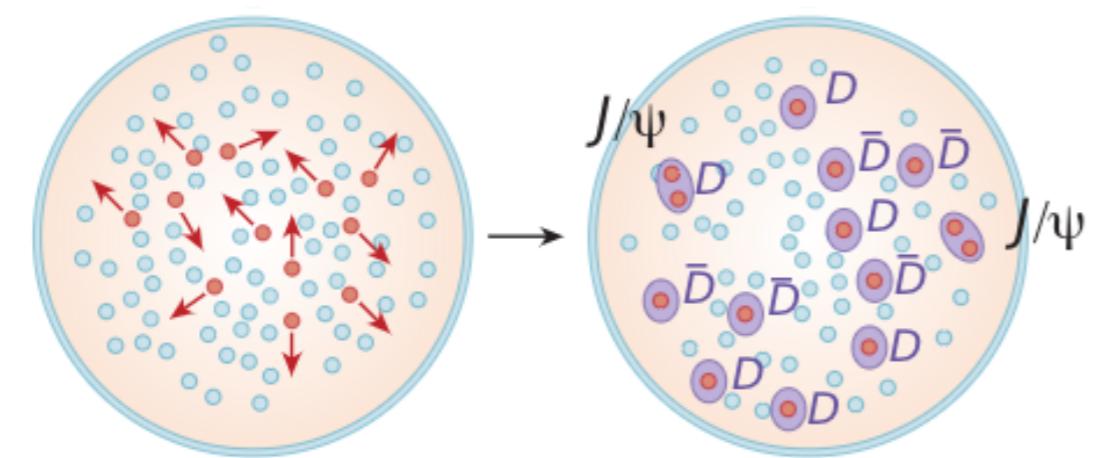
- Currently various approximations are considered *Blaizot, Escobedo '18, Yao, Mehen '18, '20*
 - Markovian limit
 - Small coupling of system and environment
 - Semi-classical transport
- Akamatsu, Rothkopf et al. '12-'20, Brambilla et al. '17-'20
Yao, Mueller, Mehen '18-'20, Sharma, Tiwari '20
Yao, Vaidya '19, Vaidya '20*

Quarkonium suppression

Open quantum system formalism for quarkonia

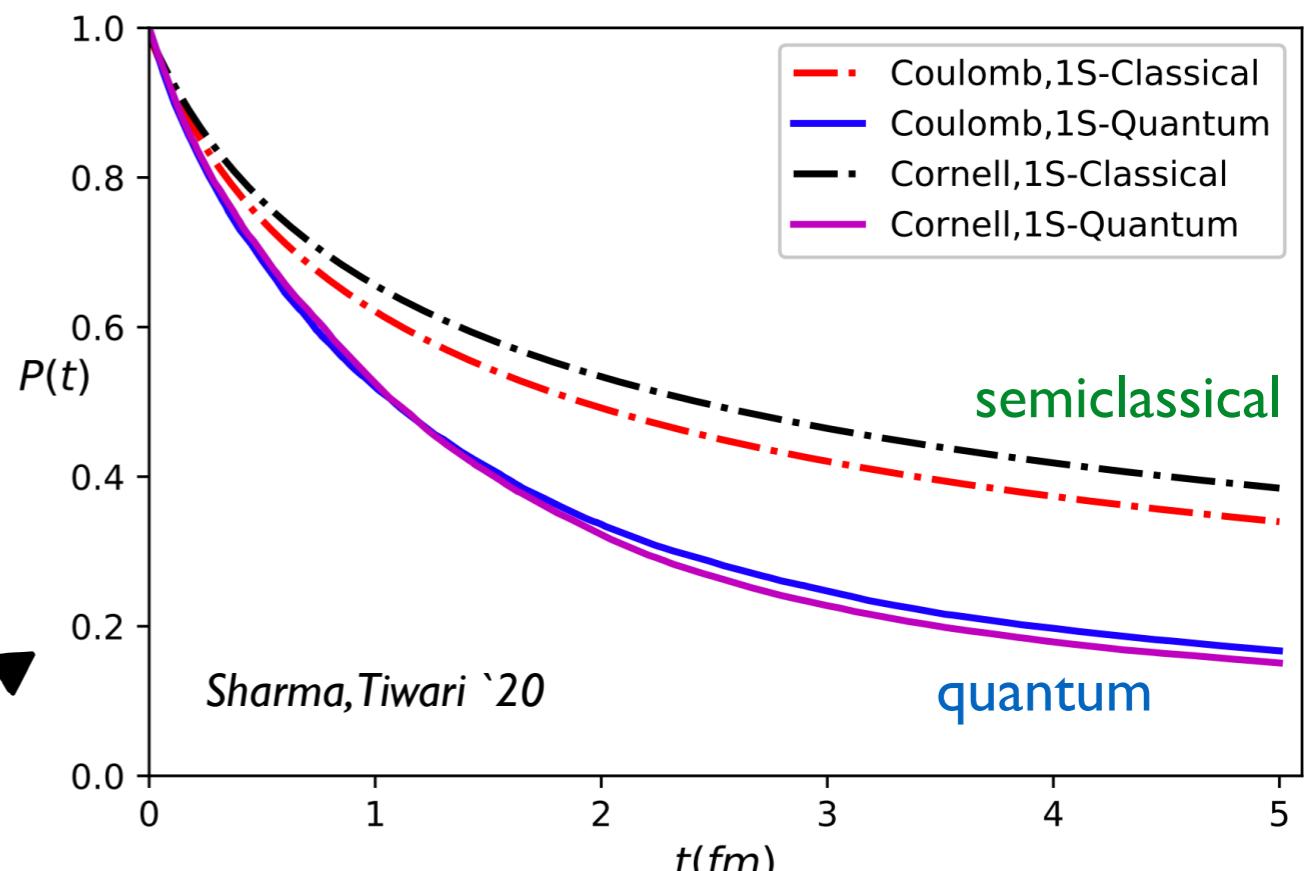
Akamatsu, Rothkopf et al. '12-'20, Brambilla et al. '17-'20
 Yao, Mueller, Mehen '18-'20, Sharma, Tiwari '20

Quarkonium production in heavy-ion collisions



NRQCD + semiclassical approach
 compared to full quantum evolution

Survival probability of the vacuum state



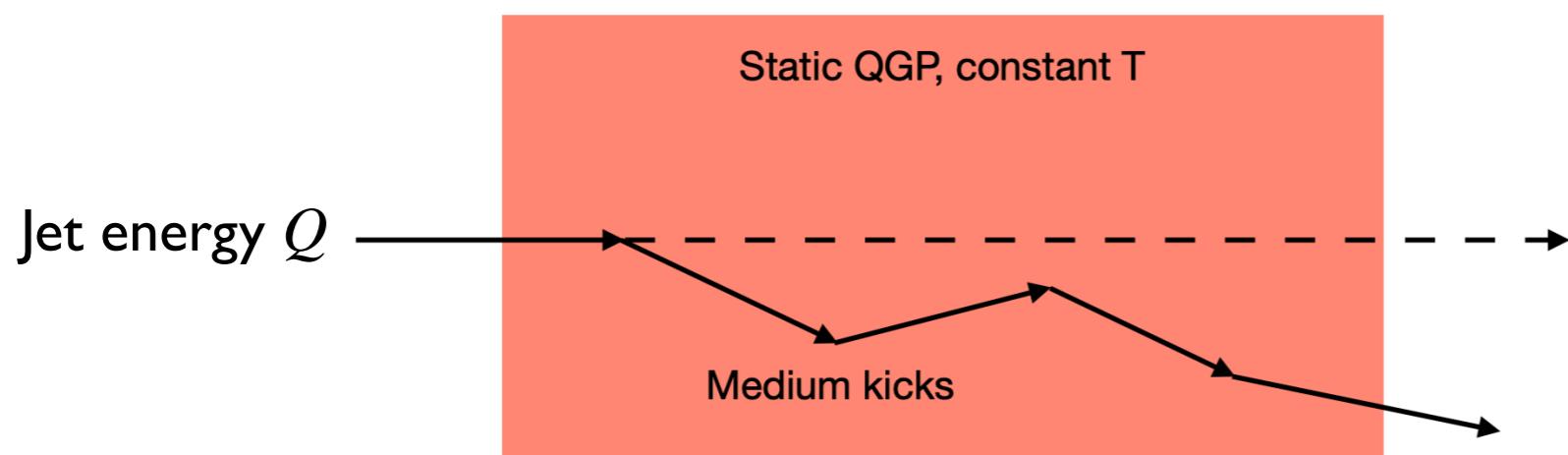
Bjorken expanding QGP $T_0 = 475$ MeV

Jet broadening

Open quantum system formalism for jets

Yao, Vaidya JHEP 10 (2020)

First steps in the direction of jet physics



Soft Collinear Effective Theory

- Forward scattering,
Glauber gluon exchange

Markovian master equation describes evolution of jet density matrix:

$$\partial_t P(Q, t) = -R(Q)P(Q, t) + \int \widetilde{dq} K(Q, q)P(q, t)$$

where the probability to be in a given momentum state is:

$$P(Q, t) = \langle Q | \rho_S(t) | Q \rangle$$

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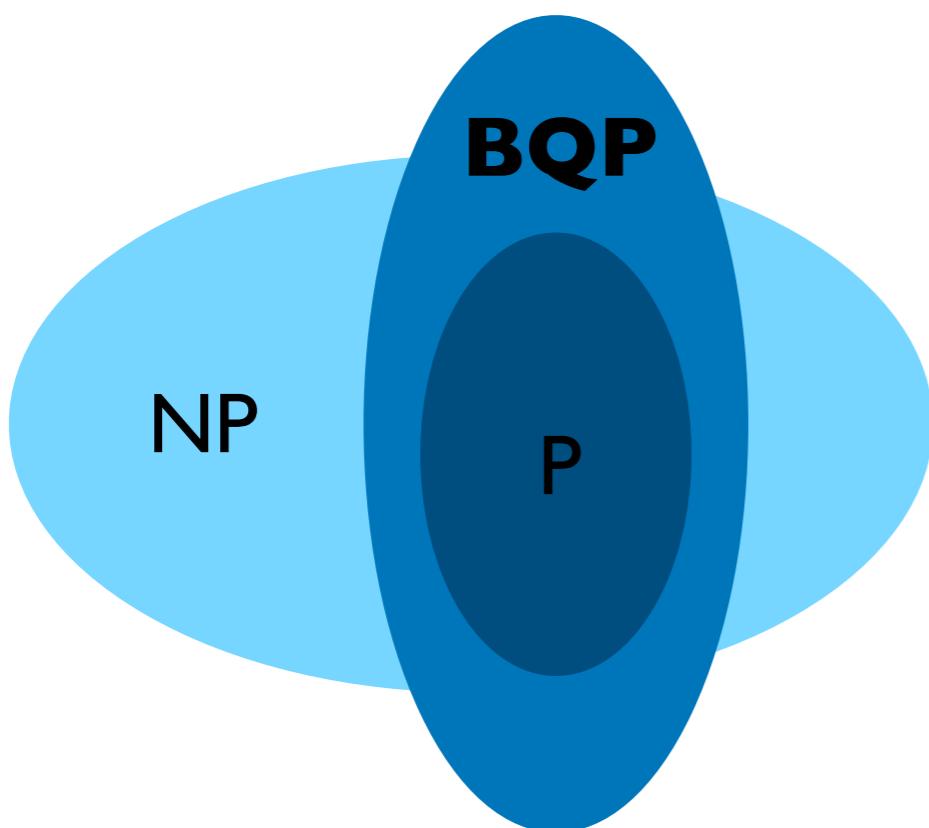
Quantum computing

Superposition and entanglement

$$|\psi\rangle = \sum_{i=1}^{2^N} a_i |\psi_i\rangle \quad \text{For } N \text{ qubits, there are } 2^N \text{ amplitudes}$$

e.g. $|\psi\rangle = a_1|000\rangle + a_2|001\rangle + a_3|010\rangle + a_4|011\rangle + a_5|100\rangle + a_6|101\rangle + a_7|110\rangle + a_8|111\rangle$

If one can control this high-dimensional space, e.g. with appropriate interference of amplitudes, then one can potentially achieve **exponential speedup** of certain computations



It is expected that quantum computers can solve some classically hard problems with exponential speedup

These include a number of highly impactful problems such as quantum simulation

Quantum devices

Superconducting circuits

IBM Q
Google Rigetti

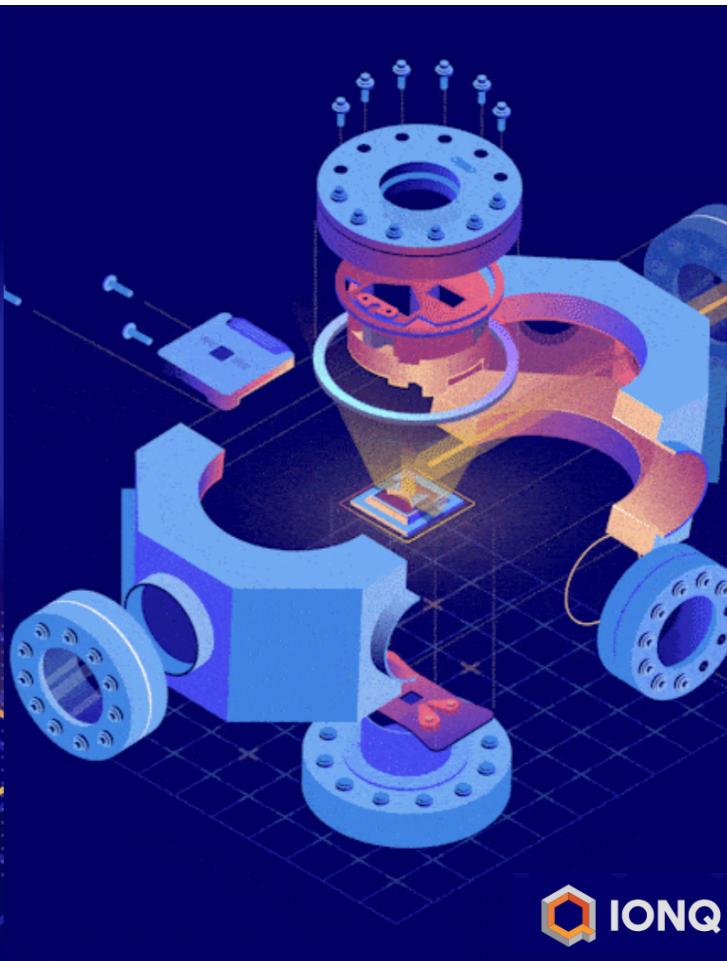
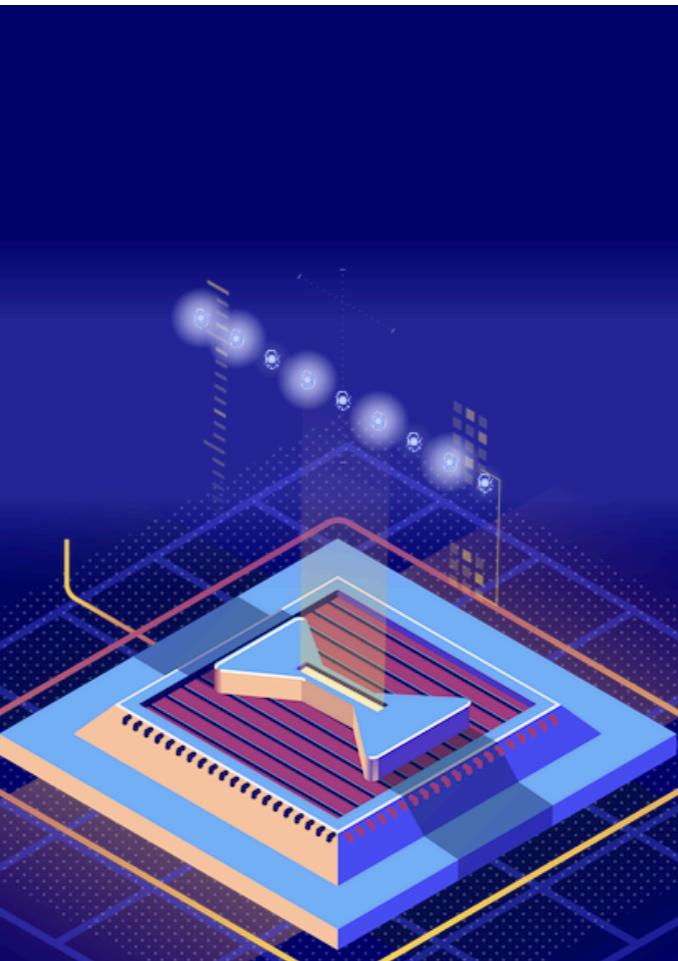
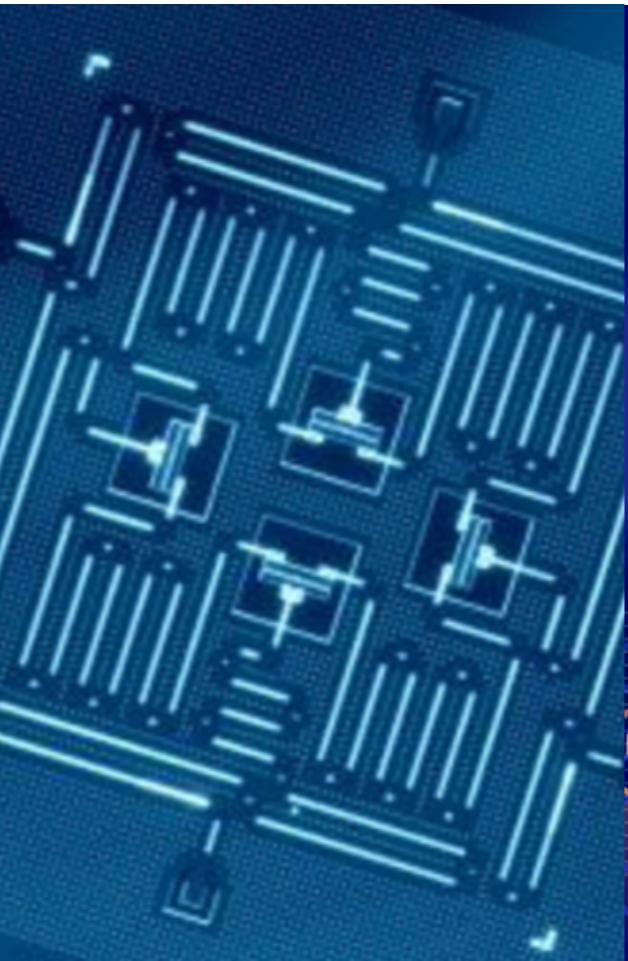
...

And a variety of others...

Trapped ions
Optical lattice
Photonics
Topological
...



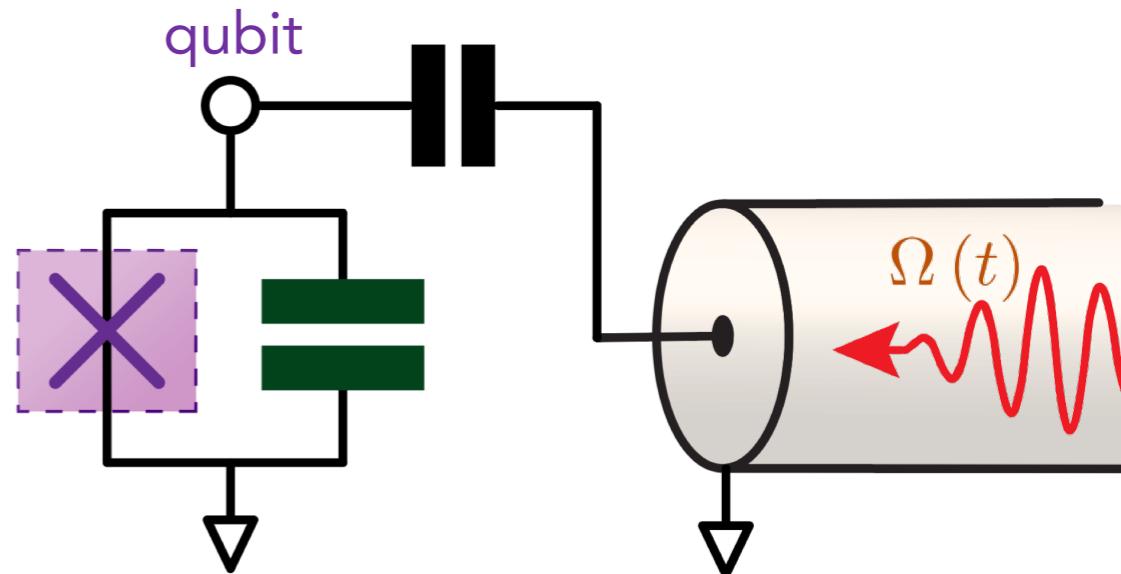
...



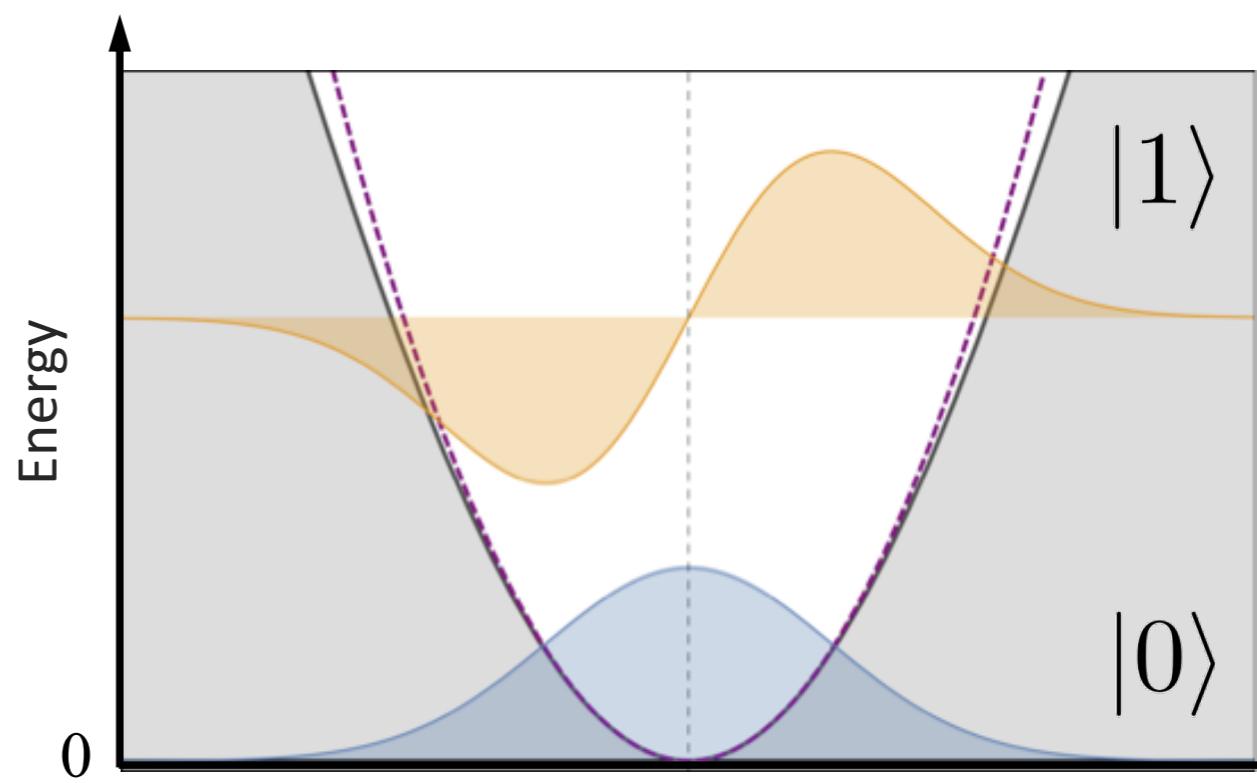
Quantum devices

Superconducting circuits

IBM Q
Google rigetti



Qubits: Nonlinear quantum oscillator
Gates: coupled microwave pulses



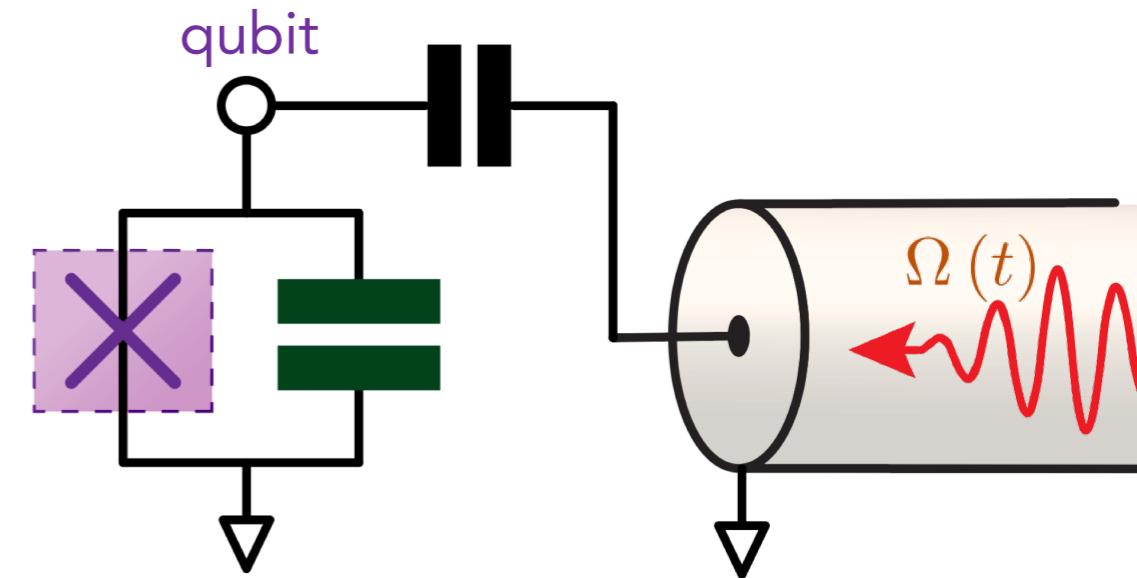
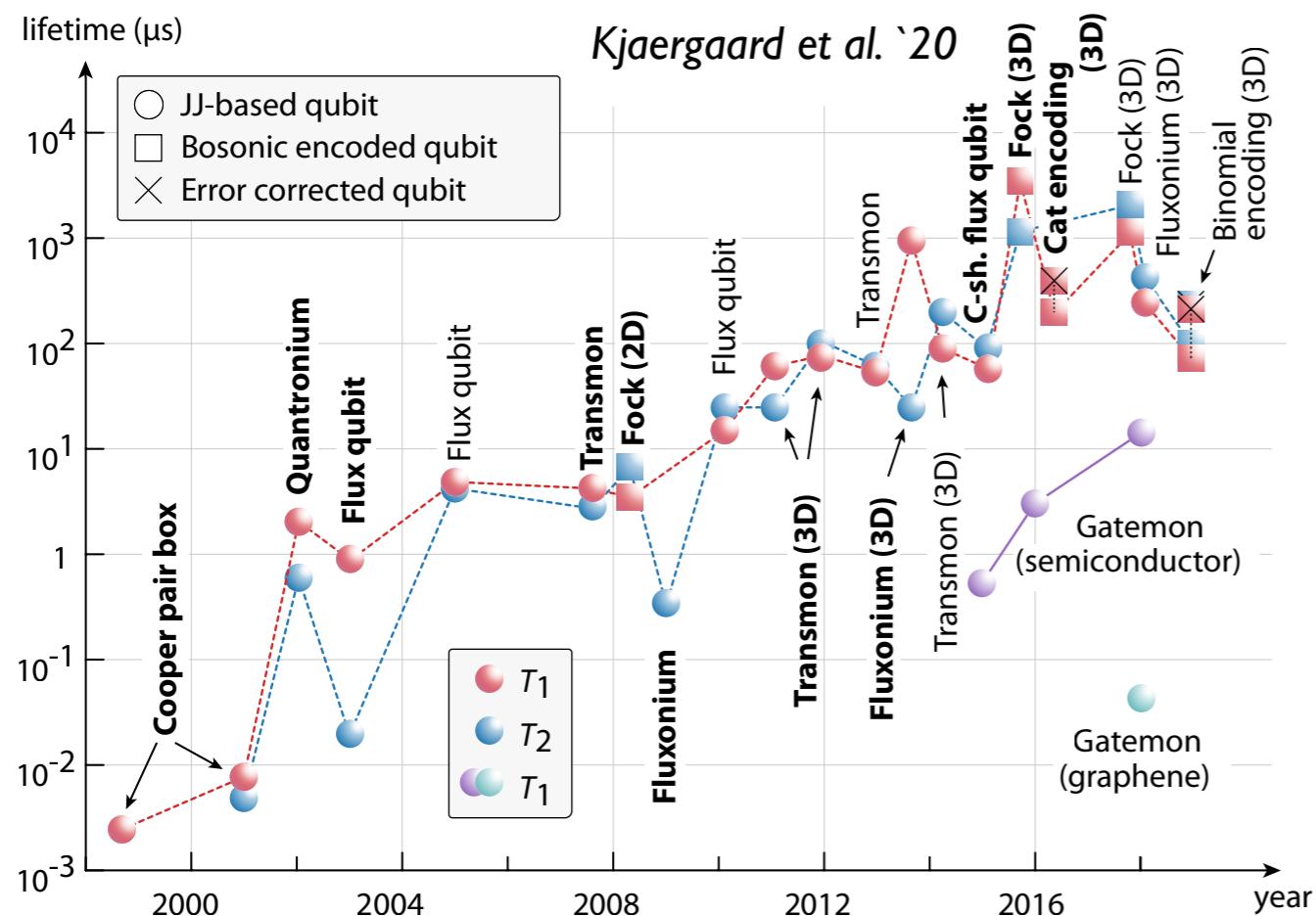
Quantum devices

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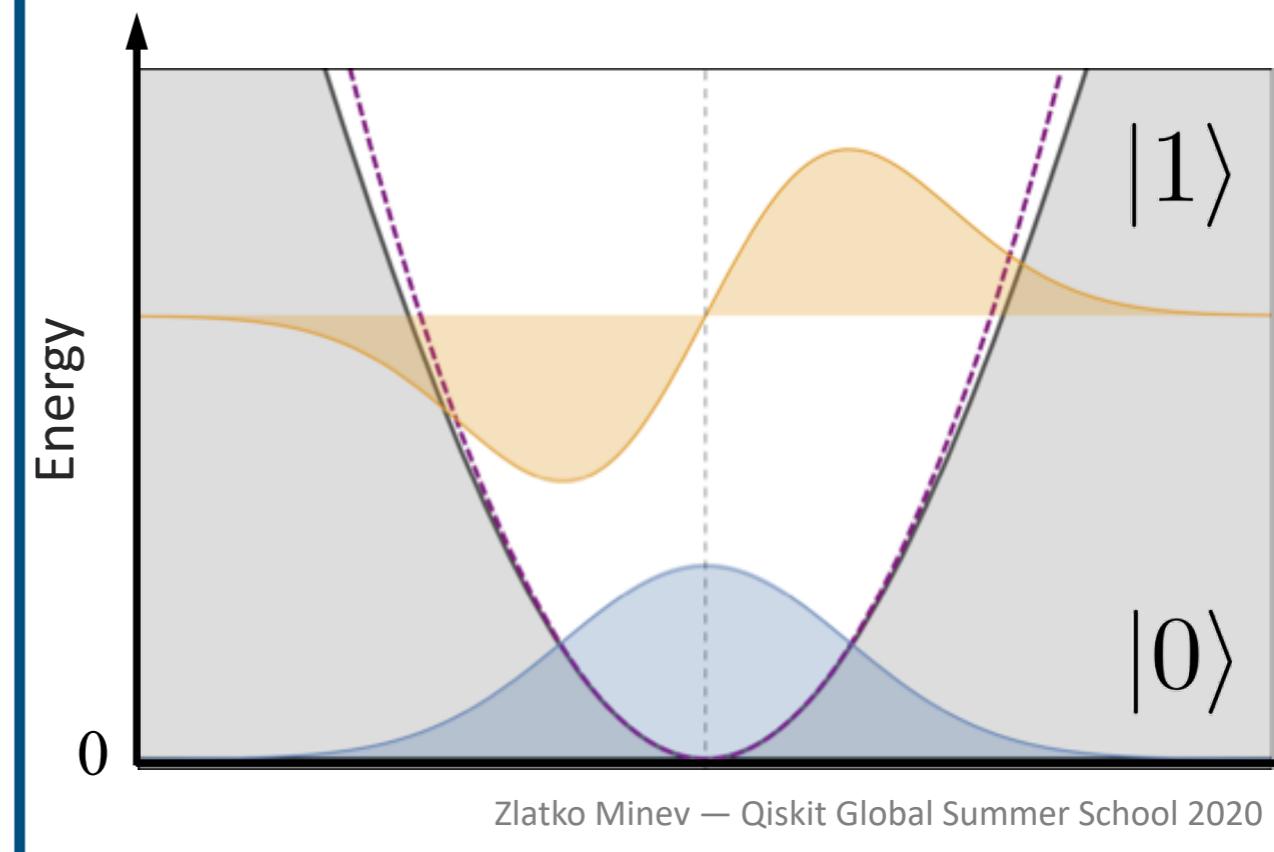
IBM Q

Google rigetti

Qubit coherence times have become $\mathcal{O}(100 \mu\text{s})$, long enough to perform $\mathcal{O}(10 - 100)$ two-qubit operations



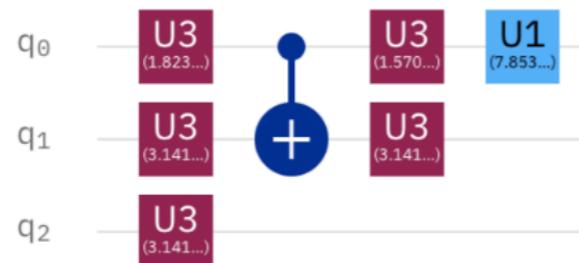
Qubits: Nonlinear quantum oscillator
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Quantum computing

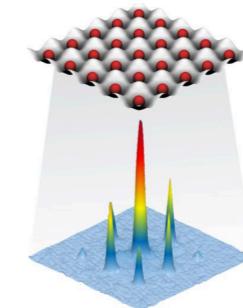
Digital quantum computers

Universal



Analog quantum computers

Application-specific



Both will likely be useful in the “near”-term

The dream: universal, fault-tolerant digital quantum computer

Shor, Preskill, Kitaev, Zoller ...

Noisy Intermediate Scale Quantum (NISQ) era

Decoherence, limited number of qubits, imperfect gates

Aim: achieve quantum advantage without full quantum error correction

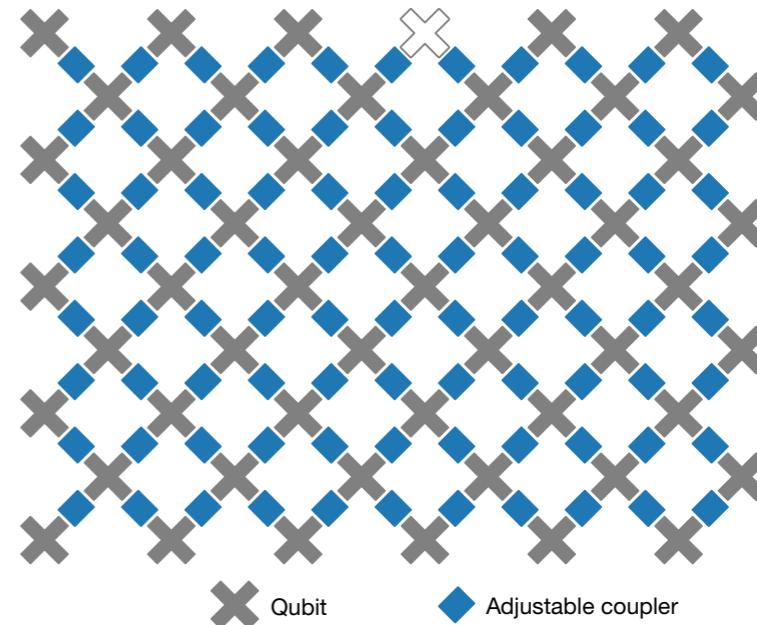
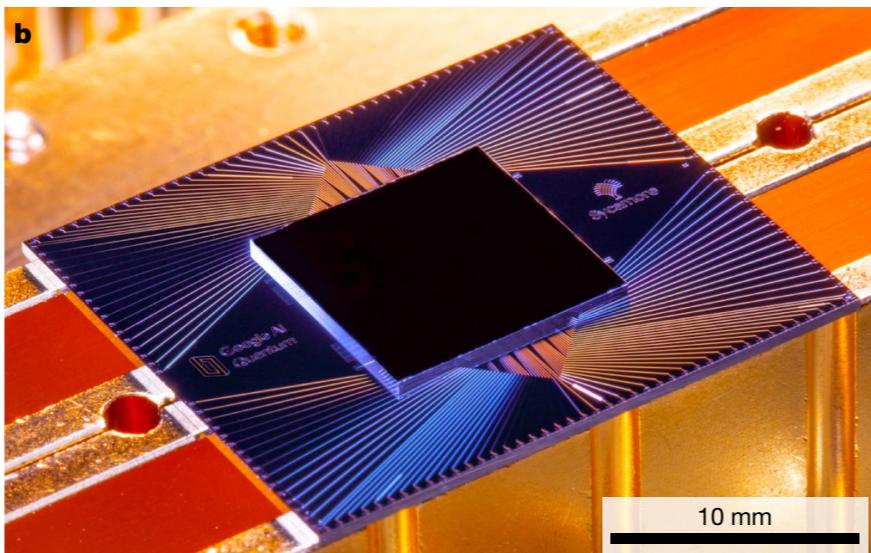
Experimentation and data analysis

Quantum supremacy

Google
Martinis et al. '19

Article

Quantum supremacy using a programmable superconducting processor



53-qubit sycamore device
99%+ gate fidelities

Algorithm: sampling of random circuits

$\mathcal{O}(10^3)$ times faster than best classical supercomputers

Quantum simulation

Feynman '81
Lloyd '96

It is exponentially expensive to simulate an N -body quantum system on a classical computer
 2^N amplitudes!

But a quantum computer can naturally simulate a quantum system

State preparation

Time evolution

Measurement



Holds great promise for particle physics

- Solve the **real-time dynamics** of QCD

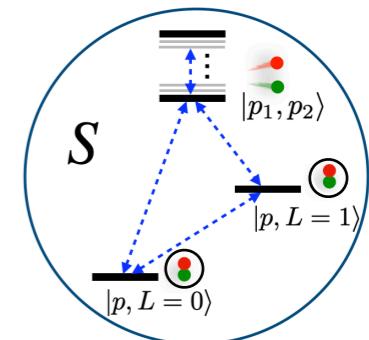
Go beyond lattice QCD limitations (static quantities — sign problem)

see e.g. Jordan, Lee, Preskill '11, Preskill '18,
Klco, Savage et al.'18-'20, Cloet, Dietrich et al. '19

Closed quantum systems

Time evolution of closed systems

- Quantum simulation of the Schrödinger equation



Evolution in time steps $\Delta t = t/N_{\text{cycle}}$

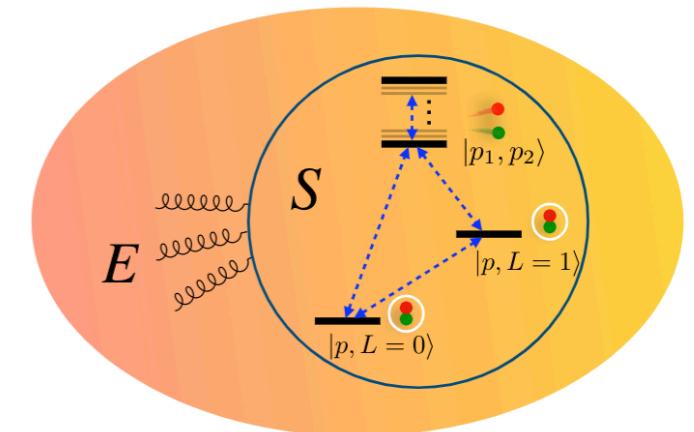
- The evolution is unitary and time reversible

→ For open quantum systems we need to introduce a non-unitarity part

Non-unitarity and time irreversible evolution

In open quantum systems, the subsystem evolution is non-unitary

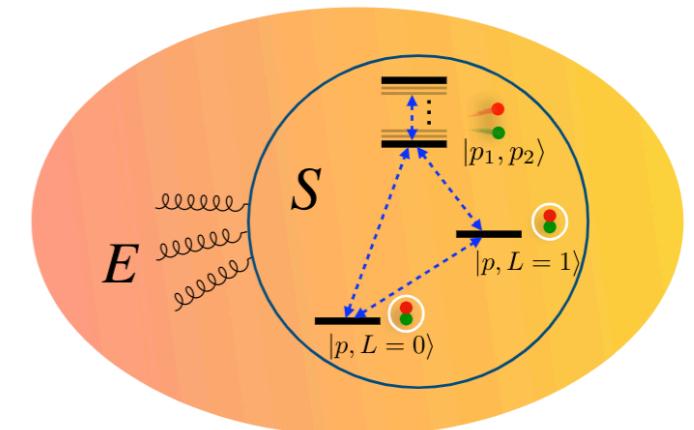
$$\frac{d}{dt}\rho_S = -i [H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right)$$



Non-unitarity and time irreversible evolution

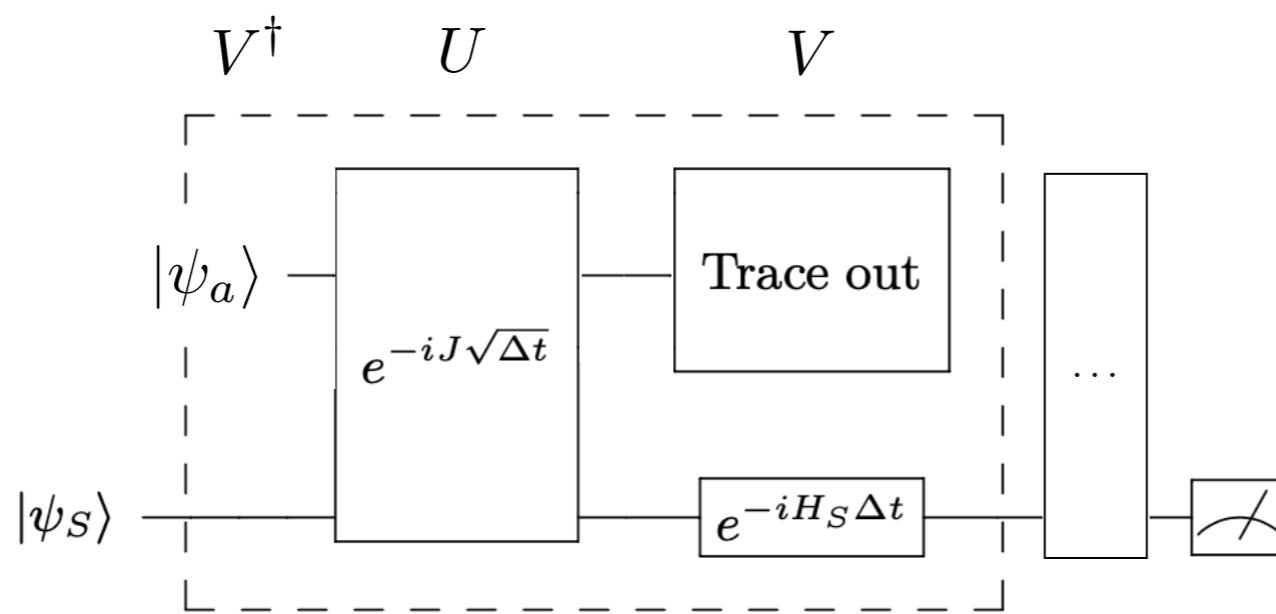
In open quantum systems, the subsystem evolution is non-unitary

$$\frac{d}{dt}\rho_S = -i [H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right)$$



The Stinespring dilation theorem

Any allowed quantum operation can be written as a unitary evolution acting on a larger space (after coupling to appropriate ancilla), and reducing back to the subsystem



$$V^\dagger V = 1 \quad VV^\dagger \neq 1$$

$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

- Evolve in time steps $\Delta t = t/N_{\text{cycle}}$

Quantum simulation of open quantum systems

Toy model setup

Two-level system in a thermal environment

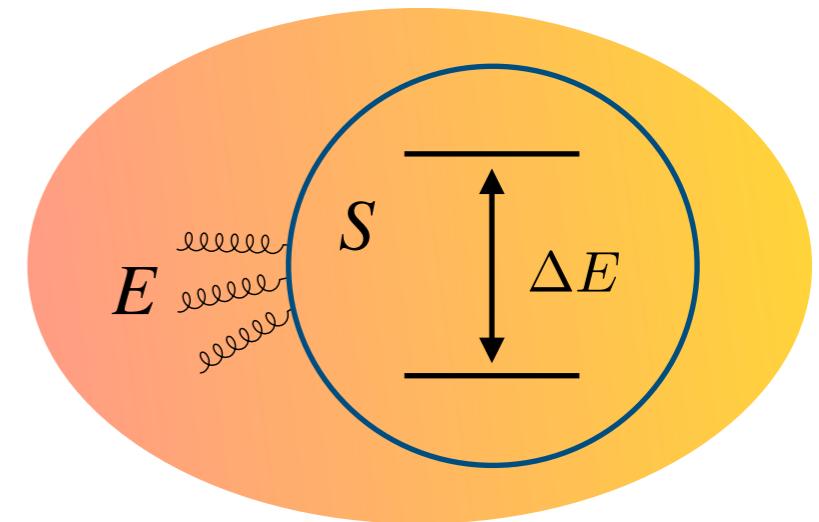
e.g. bound/unbound J/ψ , $c\bar{c}$

$$H_S = -\frac{\Delta E}{2} Z$$

$$H_E = \int d^3x \left[\frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 \right]$$

$$H_I = g X \otimes \phi(x=0)$$

Pauli matrices X, Y, Z , interaction strength g



$$\rho_E = \frac{e^{-\beta H_E}}{\text{Tr}_E e^{-\beta H_E}}$$

Quantum simulation of open quantum systems

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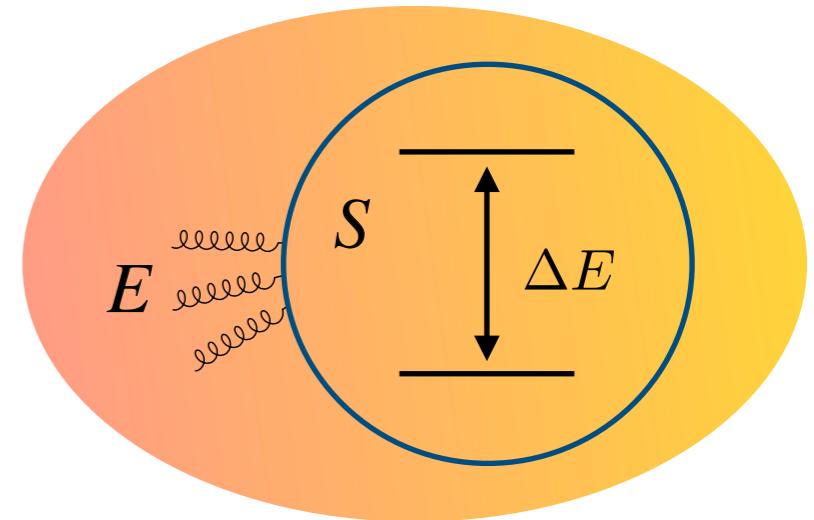
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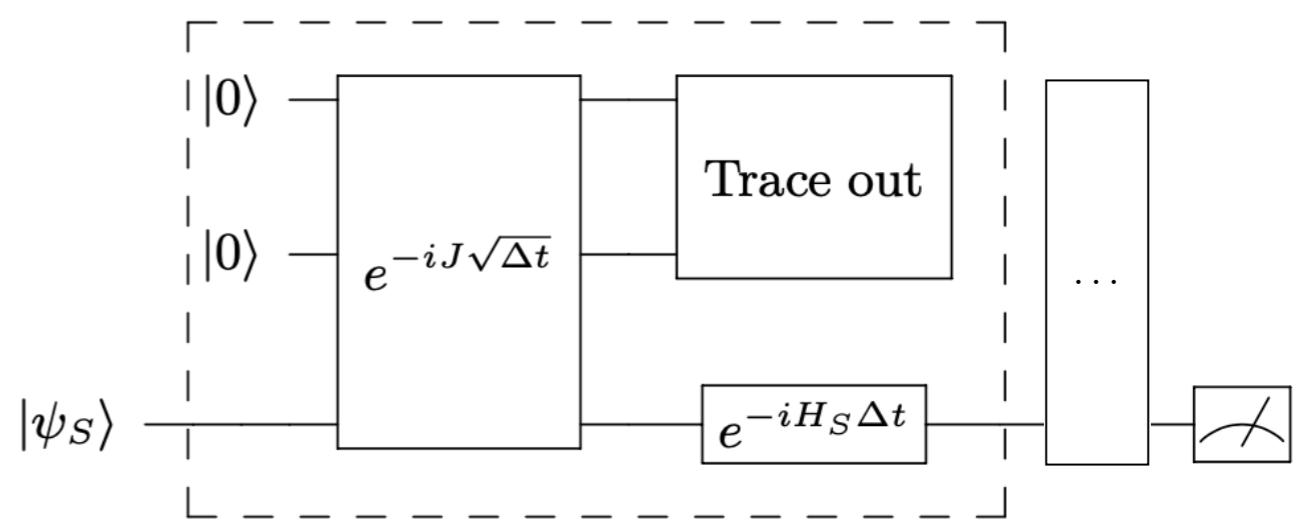
$$\rho_E = \frac{e^{-\beta H_E}}{\text{Tr}_E e^{-\beta H_E}}$$

Pauli matrices X, Y, Z , interaction strength g

Lindblad operators

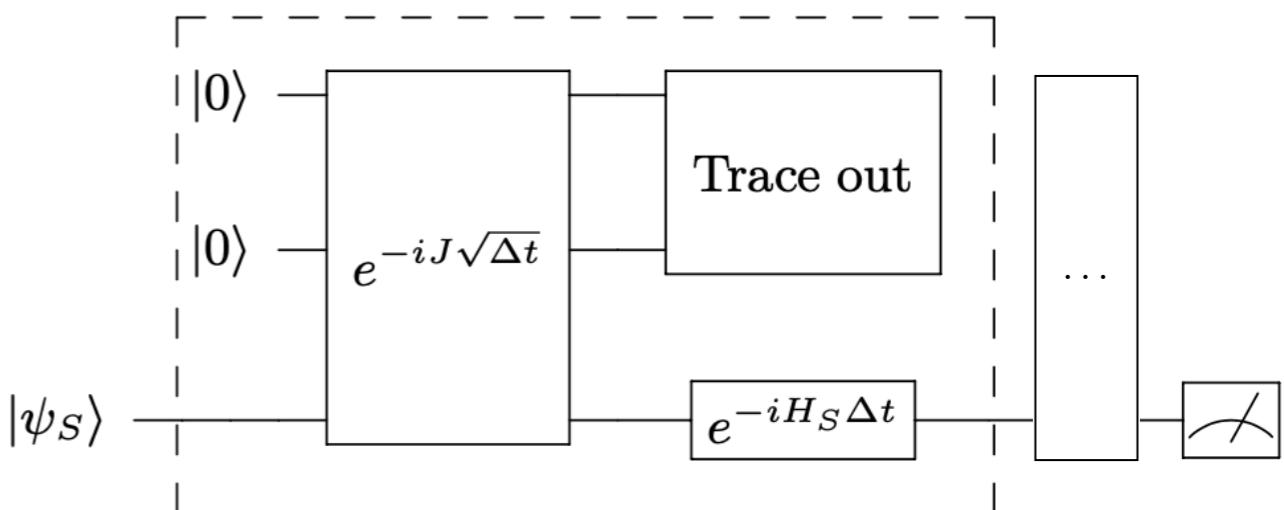
$$L_j \sim g(X \mp iY) \quad j = 0, 1$$

$$J = \begin{pmatrix} 0 & L_0^\dagger & L_1^\dagger & 0 \\ L_0 & 0 & 0 & 0 \\ L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Quantum circuit synthesis

Approximate unitary operations with a compiled circuit of one- and two-qubit gates
 Optimization problem w/unitary loss function
`qsearch` *Siddiqi et al. '20*



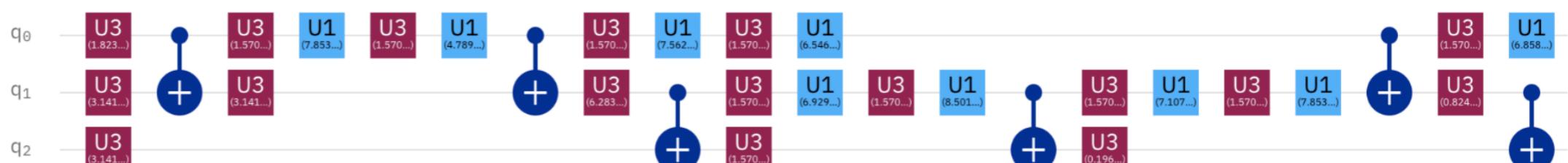
↓
10 CNOT gates/cycle

IBM Q

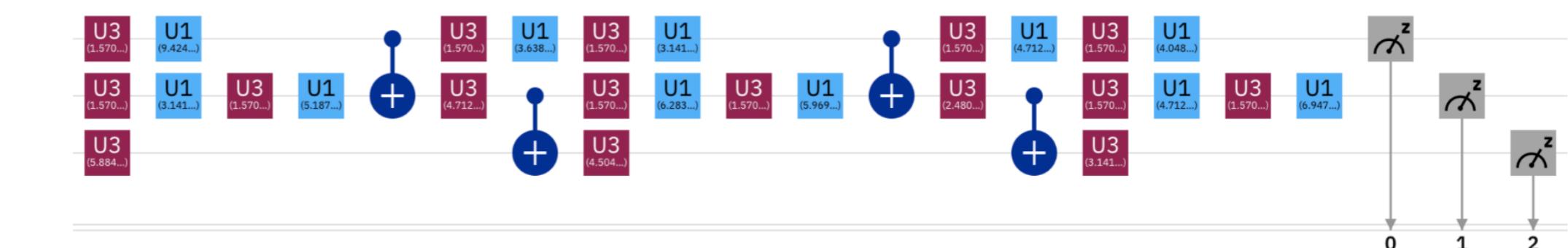
Single qubit

U1
(6.858...)

U3
(1.570...)



CNOT



Error mitigation

Readout error

Constrained matrix inversion

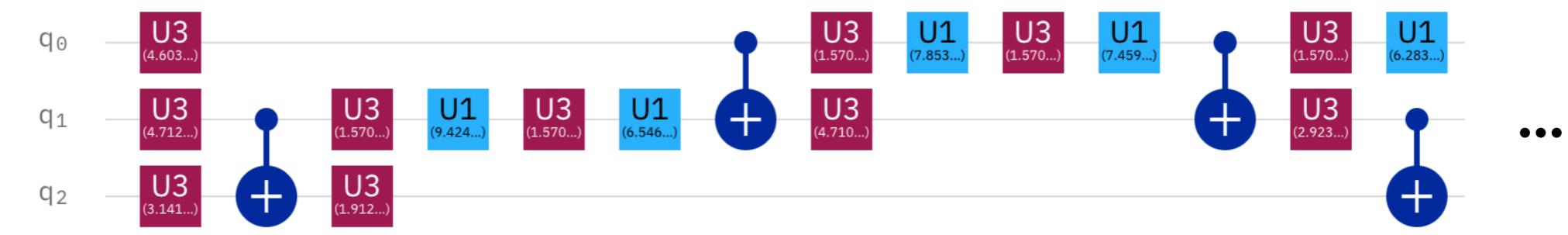
IBMQ qiskit-ignis

Gate error

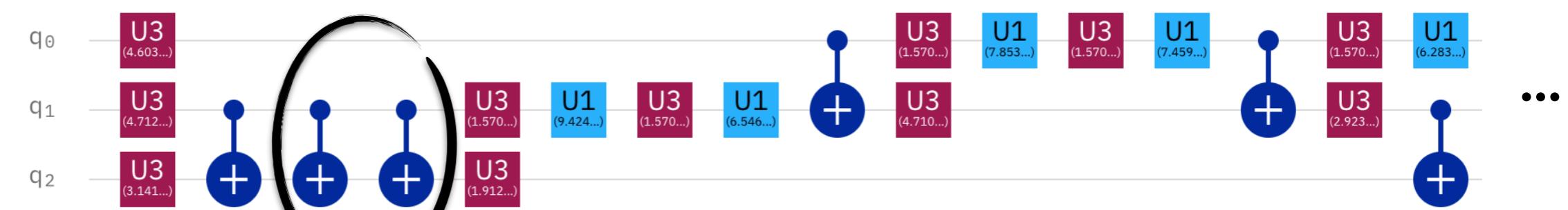
Zero-noise extrapolation of CNOT noise using Random Identity Insertions

He, Nachman, de Jong, Bauer '20

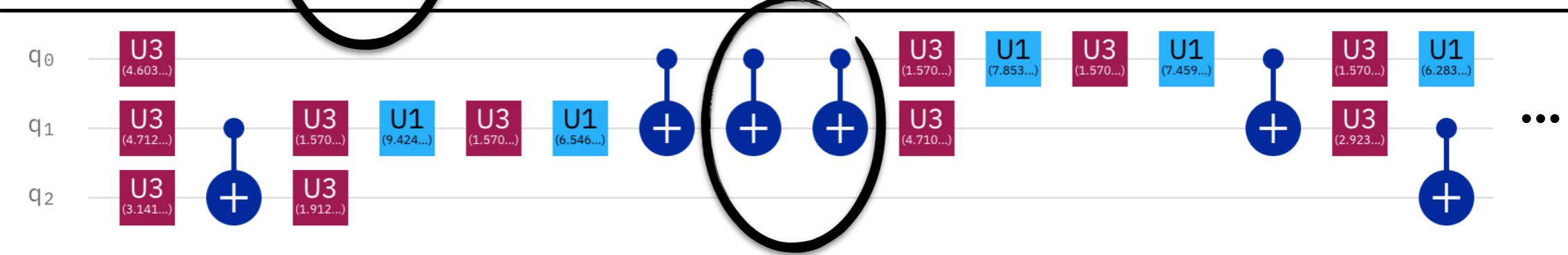
Circuit I



Circuit 2



Circuit 3



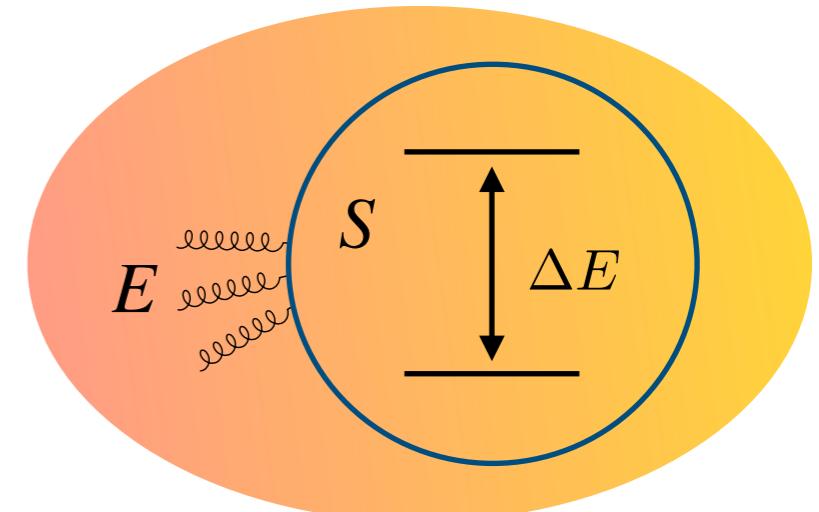
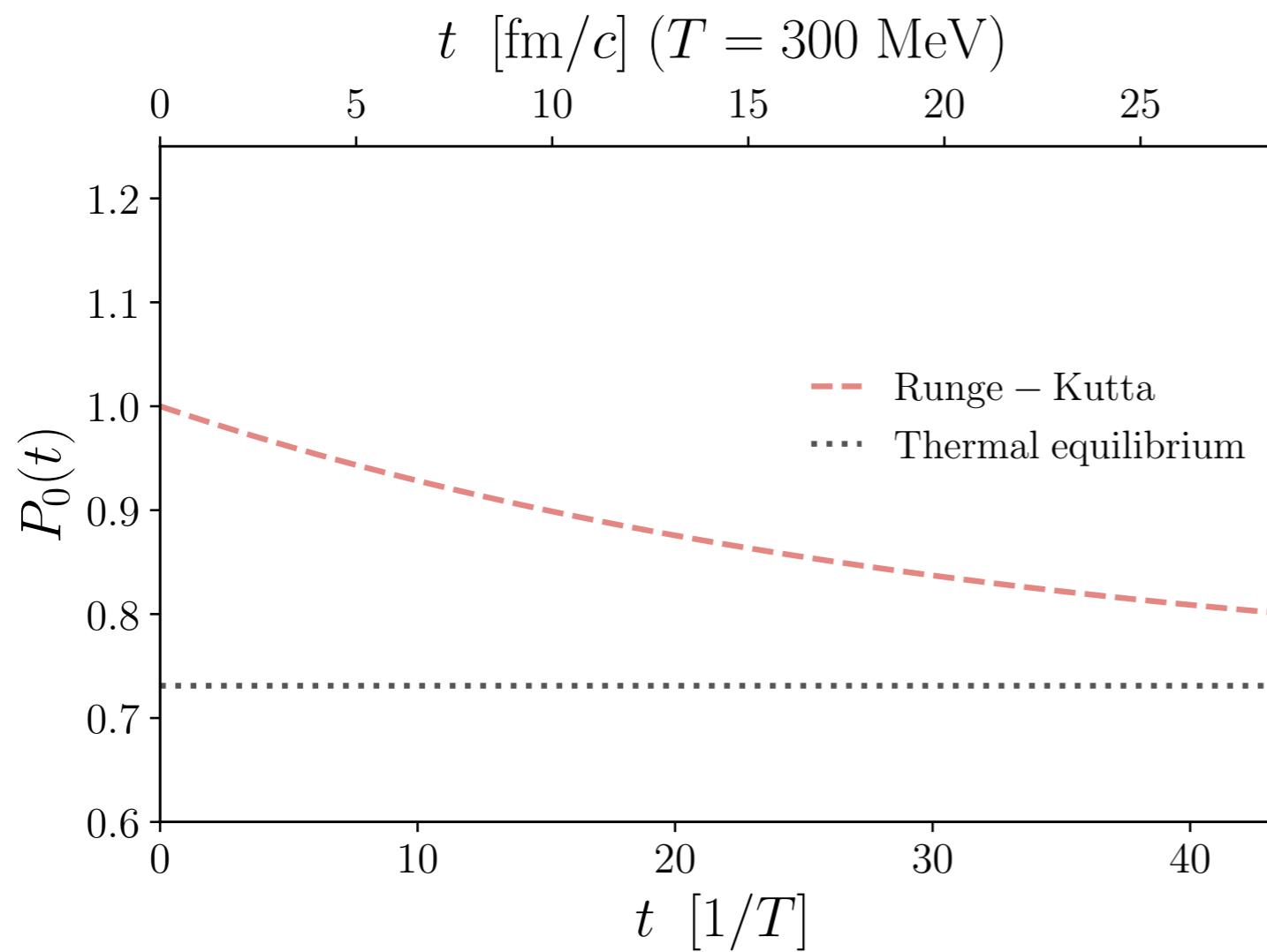
Quantum simulation of open quantum systems

arXiv: 2010.03571

Real-time evolution

$P_0(t)$ describes fraction that remains in “bound state”

Similar to t -dependent $R_{AA} = \frac{d\sigma_{AA}}{\langle N_{\text{coll}} \rangle d\sigma_{pp}}$



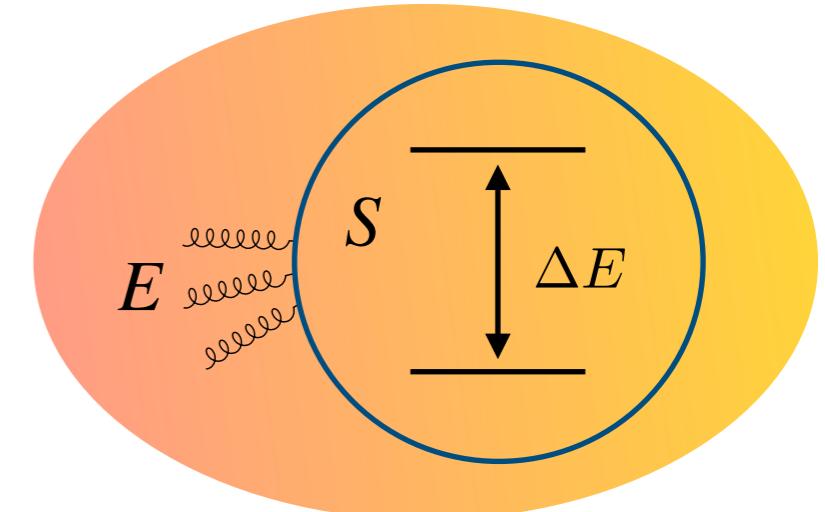
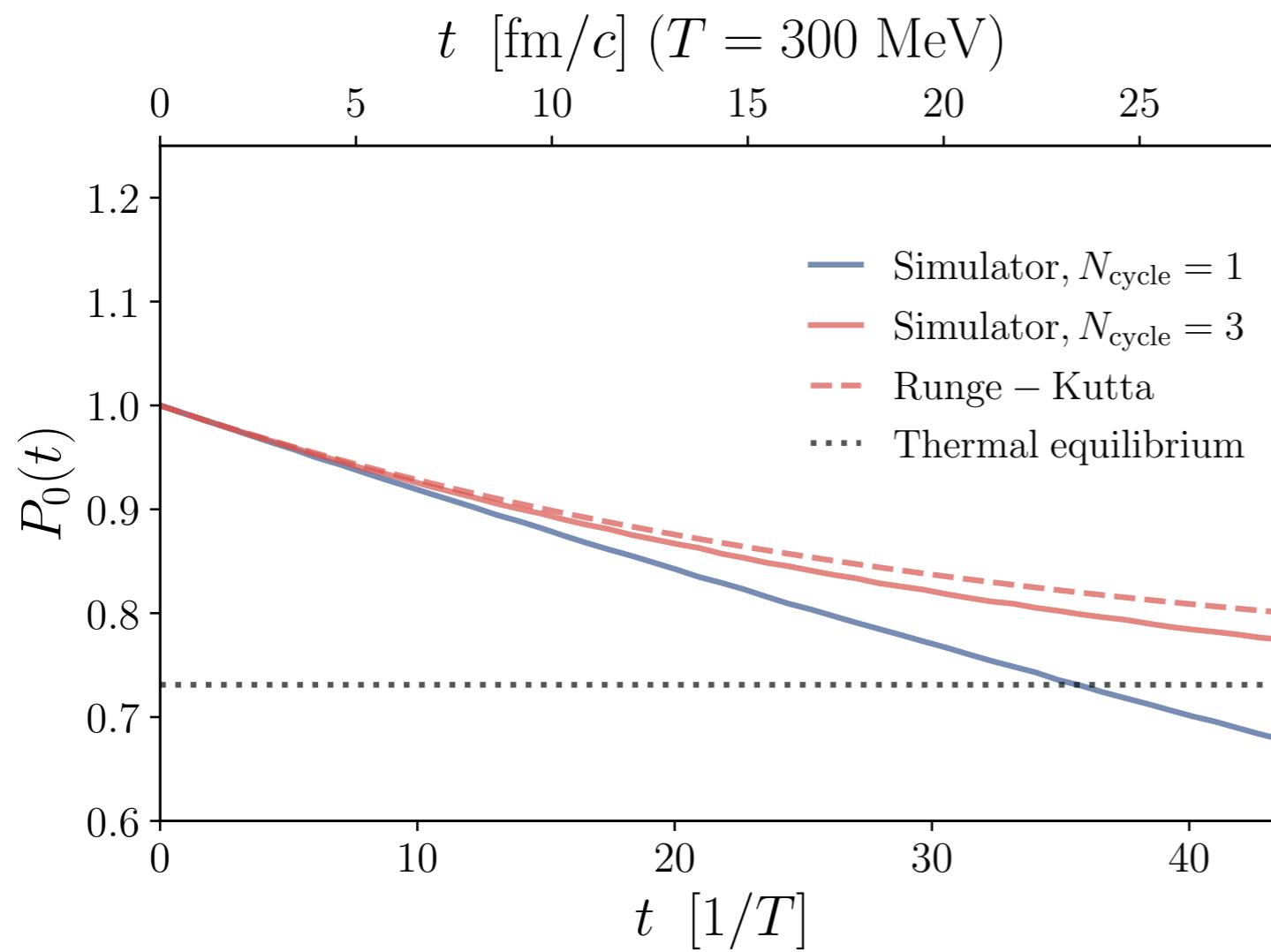
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The algorithm converges to Lindblad evolution with a small number of cycles

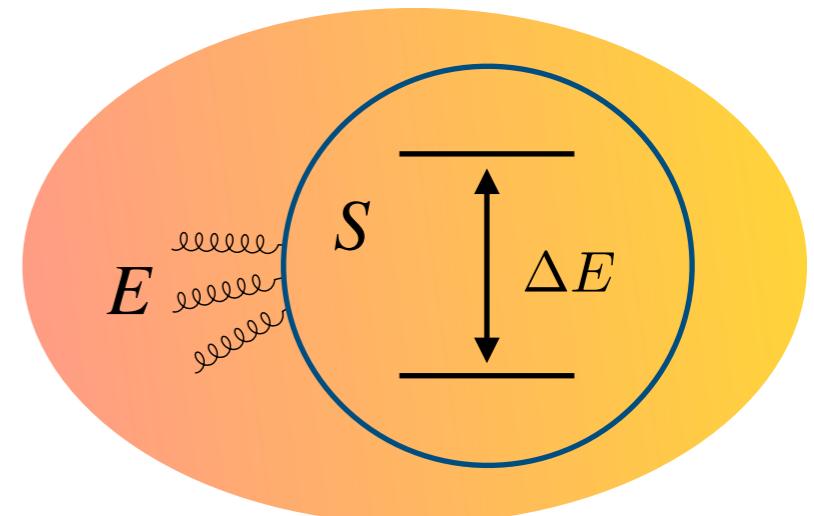
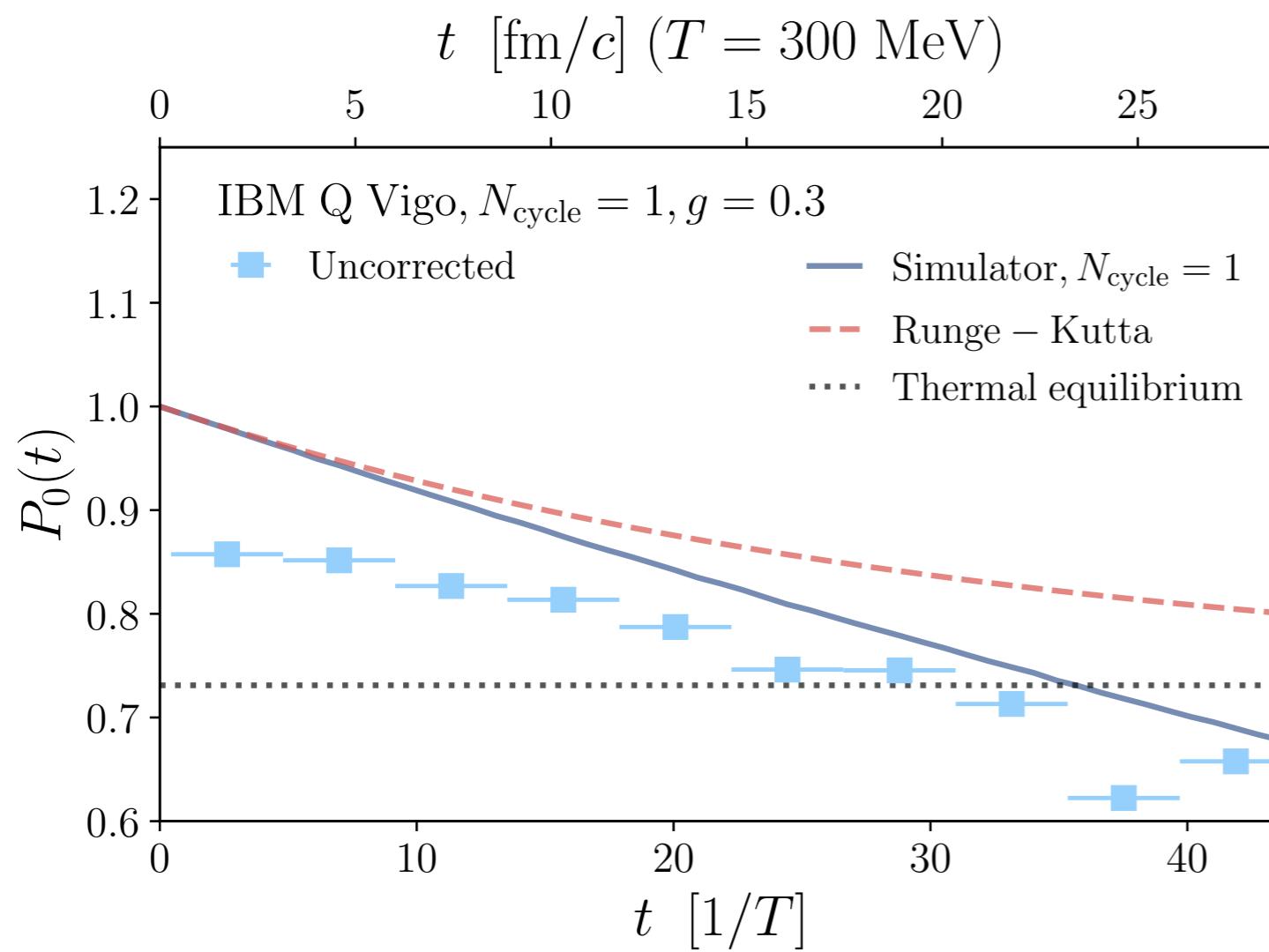
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IBM Q Vigo device

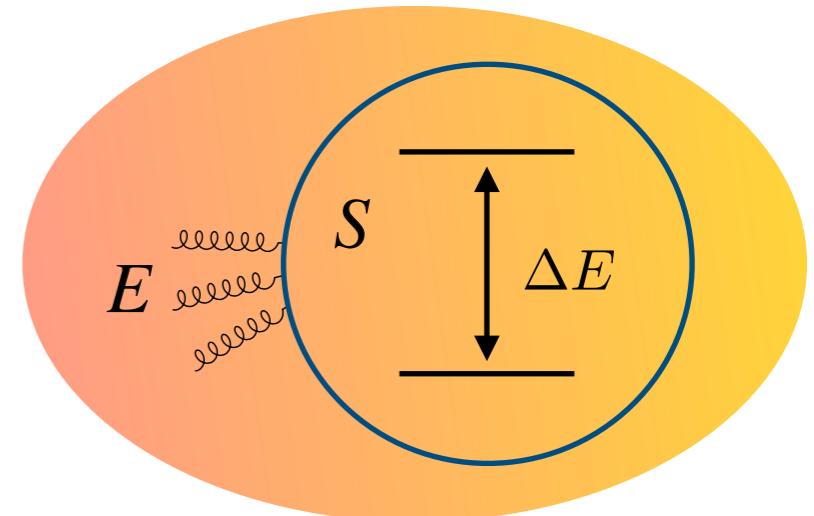
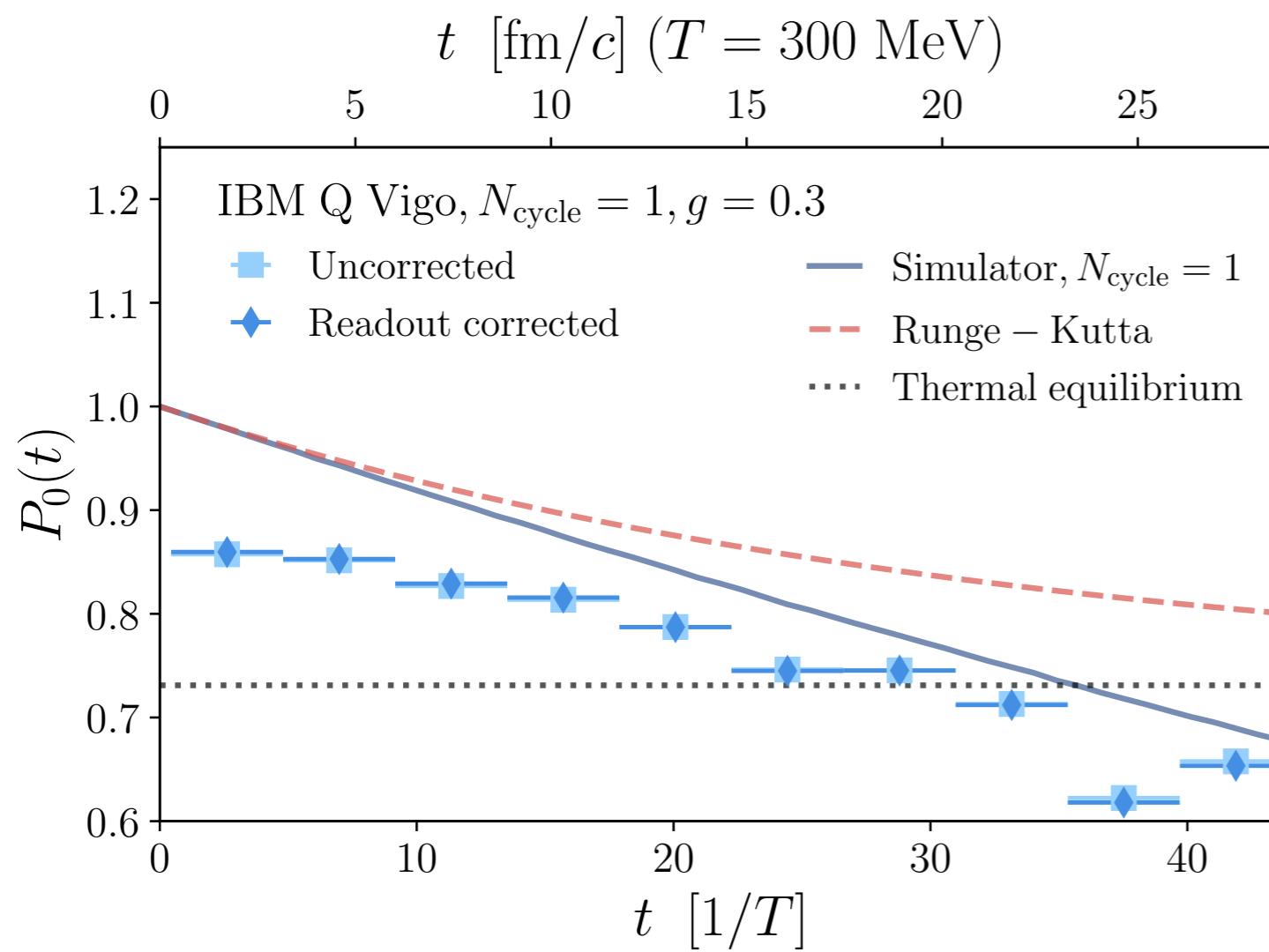
Quantum simulation of open quantum systems

arXiv: 2010.03571

Real-time evolution

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IBM Q Vigo device

Readout correction small

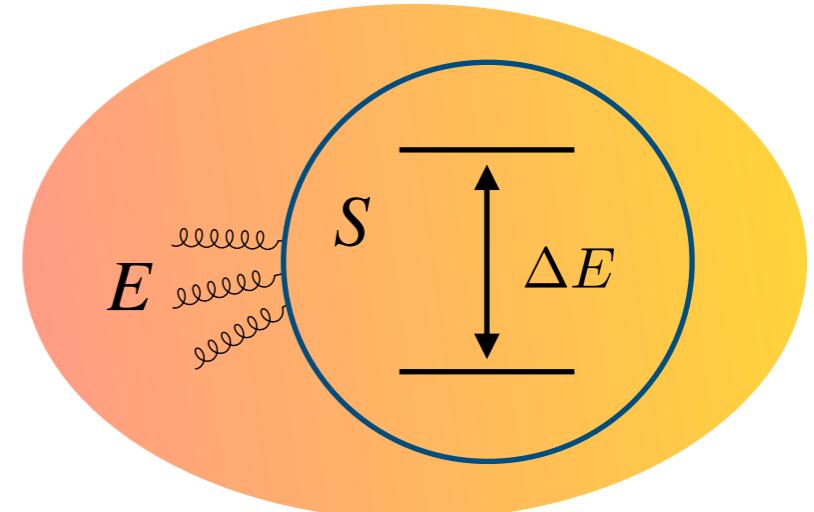
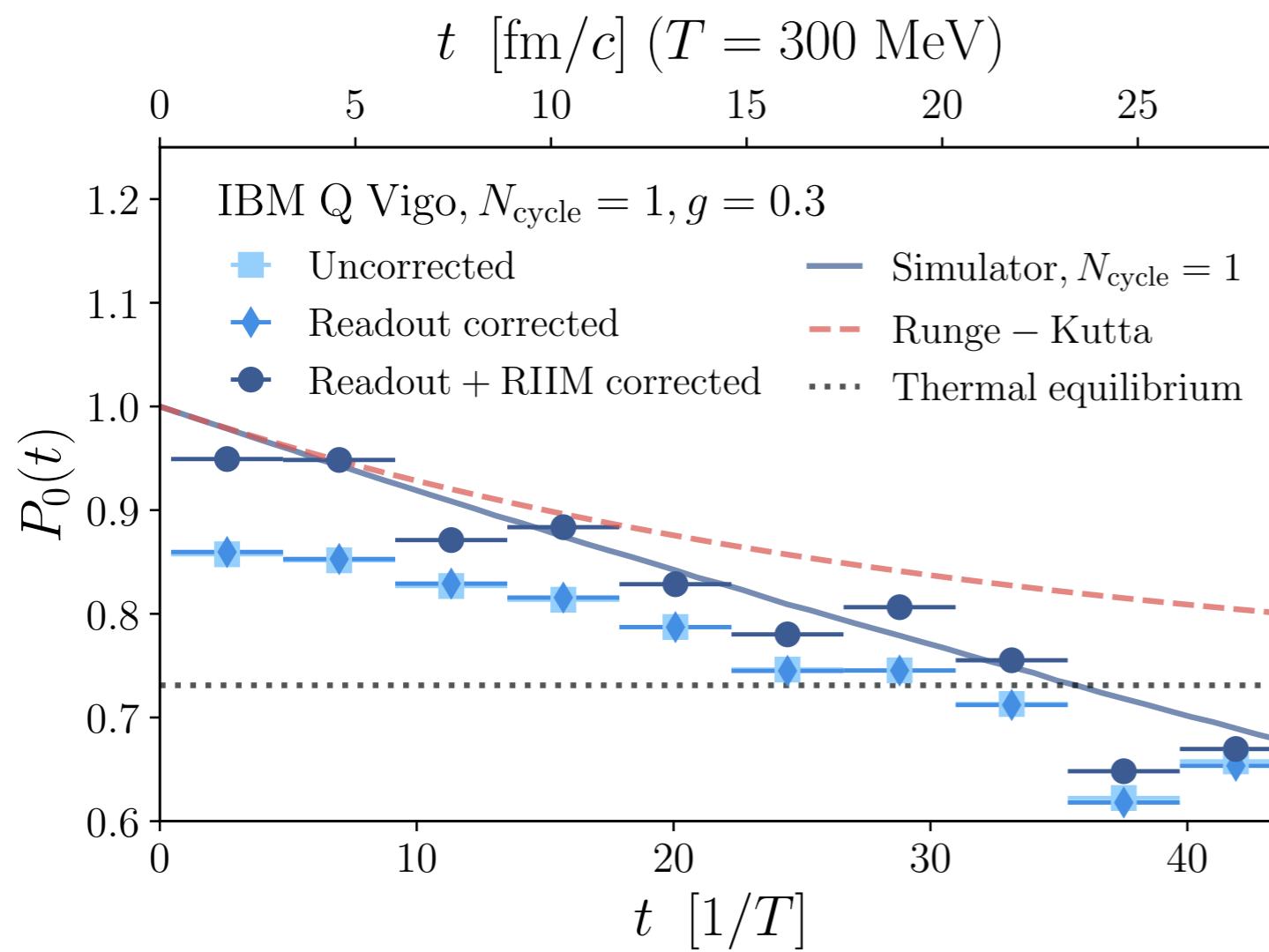
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IBM Q Vigo device

Readout correction small

CNOT gate error correction gives good agreement

Random Identity Insertion Method (RIIM)

Bauer, He, de Jong, Nachman '20

Proof of concept

Conclusions and outlook

- **Open quantum system formalism describes the real-time evolution of hard probes in heavy-ion collisions**
 - Allows to go beyond semiclassical approximations in current models
- **Proof of concept that these systems can be simulated on current and near-term quantum computers (IBM Q)**
 - NISQ era digital quantum computing
 - Recently developed error mitigation techniques
- **Future steps**
 - Extension toward QCD (jets & heavy-flavor)
 - Explore different digital/analog devices
 - Cold nuclear matter at the EIC